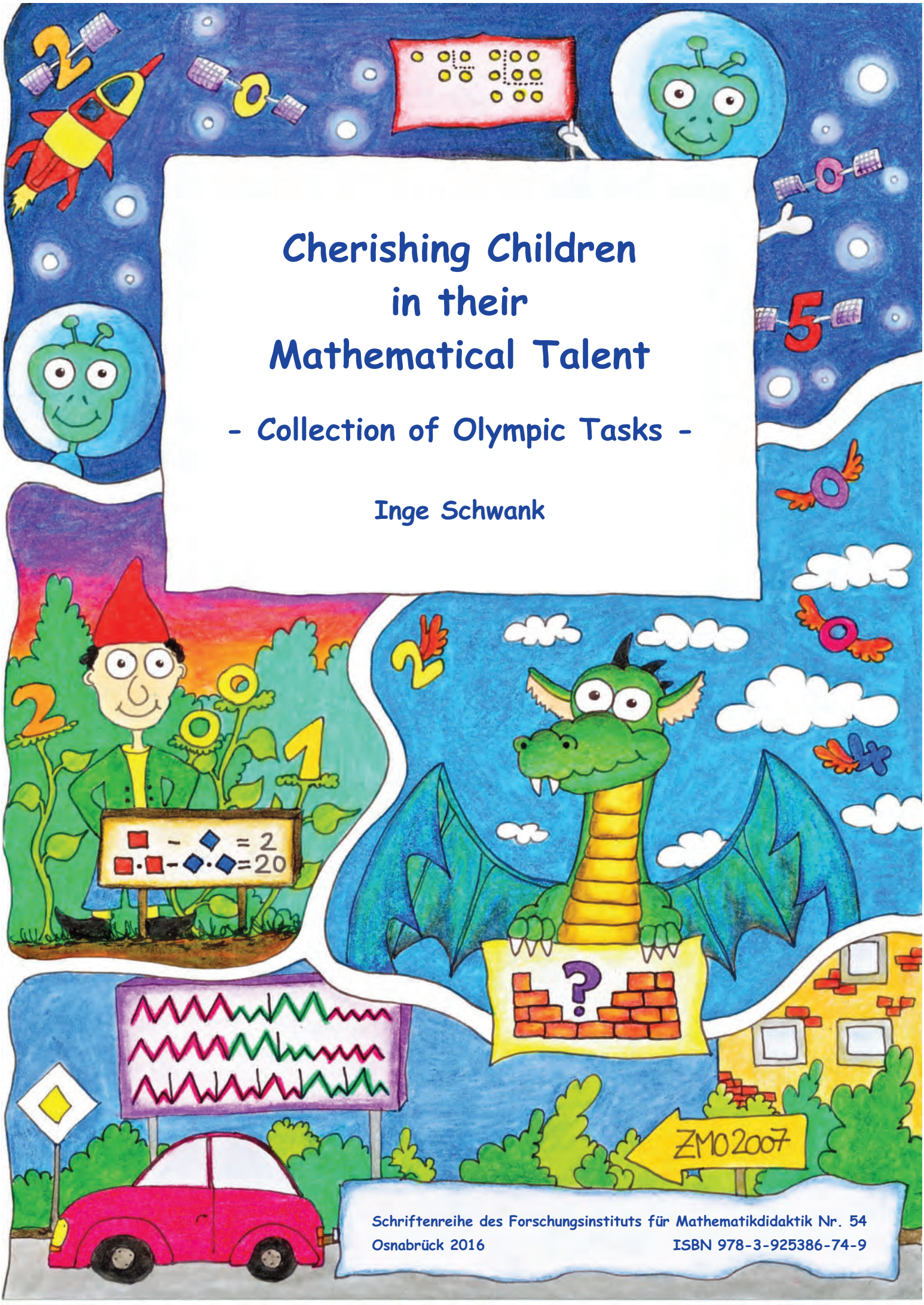

Cherishing Children in their Mathematical Talent

- Collection of Olympic Tasks -

Inge Schwank

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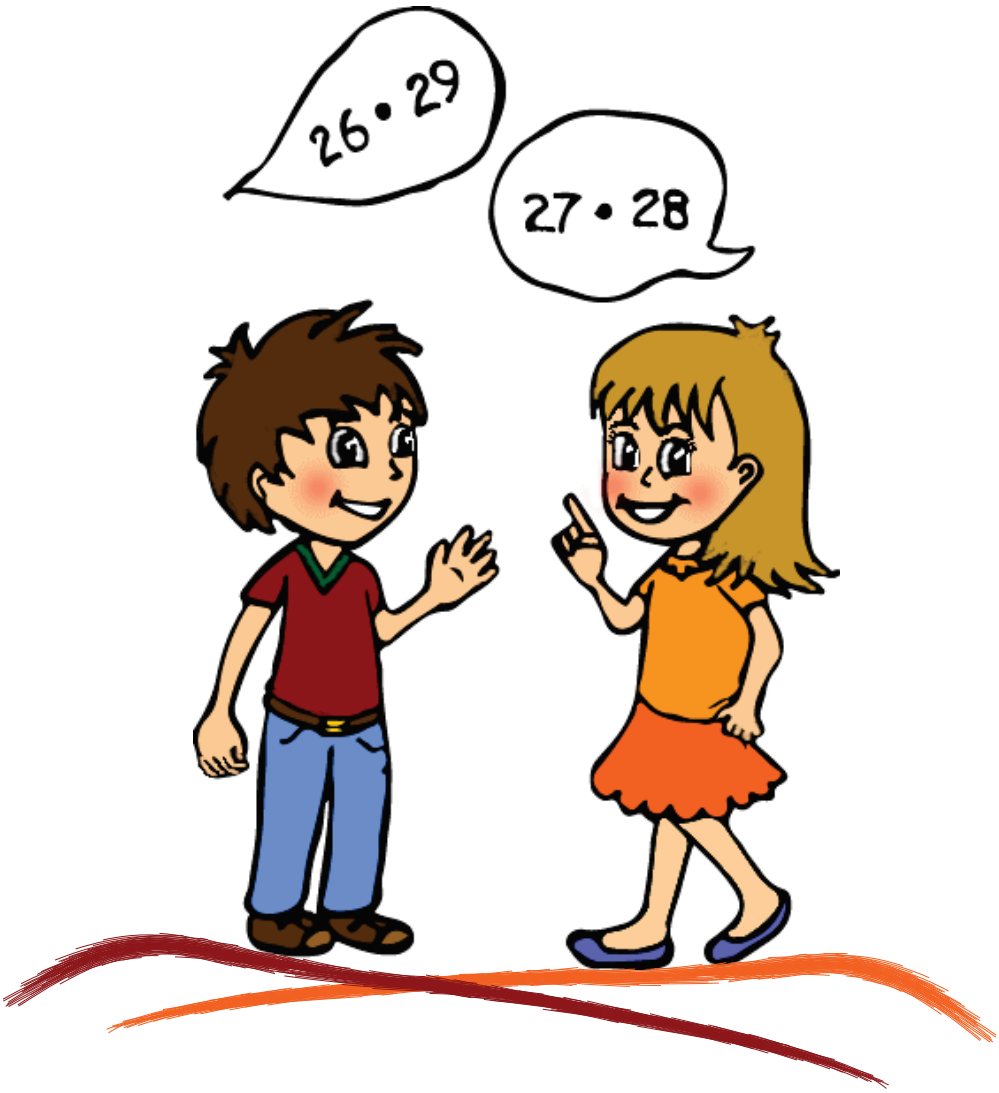

Meeting Point »Early Mathematics Education«
Scientific Direction: Prof. Dr. Inge Schwank

Schwank, Inge

Cherishing Children in their Mathematical Talent
– Collection of Olympic Tasks –

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Developed at
Meeting Point »Early Mathematics Education«

with the collaboration of many members of the ZMO-team from the years 2001-2013,
(also see the ZMO-teamlist at the end of the book);
Illustrations mainly by Burgis Hoffmann - zu Höne (also cover),
Christina Schaper & Elisabeth Schwank

Assistance with the development of the English version including the translation:
Elisabeth Schwank

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Stiftung
Stahlwerk Georgsmarienhütte

[Research Institute for Mathematics Education]
Postfach 18 47
D-49008 Osnabrück

Contents

ZMO

Zingy - Math - Olympiad

As an introduction

Statements of the ZMO-children about their attitude towards mathematics

Statements of the ZMO-children concerning their teachers

ZMO: Zingy Mathematical Olympiad, a contest for the 3rd graders in the town and district of Osnabrück, conducted by the Meeting Point »Early Mathematics Education« for 13 years. During that time, the Meeting Point has been in joint sponsorship of the Institute for Cognitive Mathematics of the University of Osnabrück and the Forschungsinstitut für Mathematikdidaktik e.V. (Reserach Institute for Mathematics Education).

Statements of the ZMO-children

We like about mathematics …

Das Mathe spaß macht und es regt die schwarzen Zelen an. Das man denken muss und Knobeln.

That math is fun and stimulates your brain cells. That one has to think and puzzle.

aller

everything

… das man ein schönes Gefühl hat, wenn man eine schwierige Aufgabe geknackt hat.

…that having solved a difficult task is a nice feeling.

- dass man manchmal auf verschiedene Art und Weise zu Lösungen kommen kann,
- dass man alleine oder im Team arbeiten kann,
- dass man Mathe jeden Tag brauchen kann.

- that sometimes one can reach a solution by different means
- that one can do everything alone or in a team
- that one can use and apply math every day

Ich finde es total spannend, fast wie ein Detektiv Rätsel und Aufgaben zu Lösen.

I find it exciting, almost like solving riddles and tasks as a detective.

Das Kopfrechnen, weil das mich richtig im Schwung bringt, Knobel-Aufgaben dann kann mein Gehirn mal richtig dampfen, und sonst gefällt mir auch alles an der Mathematik.

Mental arithmetics because that really gets me going, difficult tasks - then my brain can work itself up, and also I lika everything else about mathematics.

Mathe ist spannend, abwechslungsreich mit guten Aufgaben. Mathematik braucht man fürs Leben und für den normalen Alltag. Mathe bietet leichte, mittel und schwierige Aufgaben. Mathe fördert das Gehirn. Und außerdem macht Mathe einfach Spaß.

Math is exciting and offers great variation with good tasks. One needs mathematics for life and for everyday business. Math offers easy, medium and difficult tasks. Math educates the brain. And also, math is just fun.

dass wir immer etwas Neues lernen. Mathematik ist für das eigene Leben, für die Zukunft und einen guten Beruf wichtig. Dividieren, subtrahieren mal nehmen oder addieren - Rechnen macht natürlich Spaß, wenn du Mathe gerne hast.

that we constantly learn something new. Mathematics is important for one's own life, for the future and for getting a good job. Of Course dividing, the subtracting, multiplying and adding is fun if you like math.

dass es viel Spaß macht. Man kann nicht nur rechnen, sondern auch zeichnen, messen oder etwas wiegen. Ich finde es gut, wenn man verschiedene Rechenwege ausprobieren kann.

that it is a lot of fun. It is possible to not only calculae but also to draw, to measure or to weigh something. I like it when one can try different ways of calculating something.

das wir immer wieder mit noch mehr und noch größeren Zahlen rechnen können.

that we can again and again calculate with more and bigger numbers.

Die guten Noten, Die schweren Aufgaben (sehr sehr sehr schwer beispiel 373 • 591 = 220443)

The good grades. The difficult tasks (very, very, very, very difficult such as 373 x 591 = 220443)

that there are so many nice tasks.

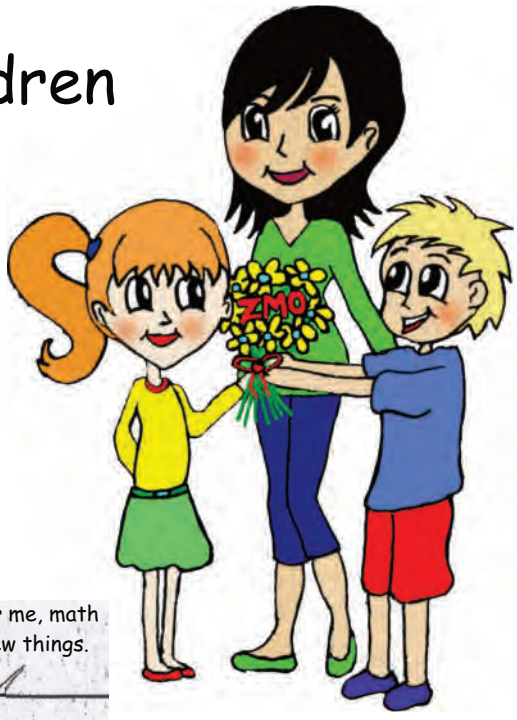
das es so viele schöne Aufgaben gibt.

Because math is cool

Weil Mathe cool ist

Statements of the ZMO-children

Concerning their teachers …



Wir finden an Mathematik gut ...
We like about mathematics...

die Lehrerin. Wir lernen mit Zahlen zu rechnen, damit wir im Leben gut klar kommen.

the teacher. We learn calculating with numbers so that we get along well in life.

Wir finden an Mathematik gut ...
We like about mathematics...

Ich habe eine tolle Lehrerin und mir macht Mathematik sehr viel Spaß. Ich lerne viele neue Sachen.

I have a great teacher and for me, math is a lot of fun. learn lots of new things.

Wir möchten an der ZMO-Hirnsportrunde teilnehmen, weil ...
We want to participate at the ZMO-BrainTrain-Round, because...

Uns Mathe Spaß macht, weil unsere Klasse recht gut rechnen kann, weil unsere Mathelehrerin uns gut trainiert hat und weil wir es ohne sie gar nicht wüssten.

We like doing math, because our class is rather good in calculating, because our math teacher trained us very well and because without her wouldn't know that.

Wir finden an Mathematik gut ...
We like about mathematics...

Das der Lehrer so nett ist, Das der Unterricht ein bisschen lustig ist, abwechlungsreich

...That the teacher is so nice, That classes are a little bit funny and offer great variation

Wir finden an Mathematik gut ...
We like about mathematics...

dass unsere Lehrerin dafür sorgt, dass der Unterricht nie langweilig wird und gibt uns immer neue Aufgaben zum rechnen.

that our teacher makes sure that classes are never boring and she always gives us new tasks to calculate.

A. Introduction

Mathematics has beauty and romance.
It's not a boring place to be, the mathematical world.
It's an extraordinary place; it's worth spending time there.

~ Marcus du Sautoy ~

The social perception of and interest in mathematics goes beyond simply regarding it as a useful tool for science and daily life. For a long period of time, mathematics has had to struggle with its widespread, emotionally highly charged reputation for being a difficult and tedious, yet necessary subject. Interestingly, a certain change of mind seems to emerge in the recent years. For instance, the Deutsche Mathematiker Vereinigung [German Mathematical Society] refers to a representative study published in 2010 (see link below) revealing that mathematics ranks second among the favorite subjects of German students from year 5 upwards. Furthermore, 68% of the adults also interviewed during this study reported to enjoy dealing with math problems in daily life. This can be seen as an indication of a change of perspective away from prejudiced, negatively connoted thinking to viewing and enjoying mathematics with an open and interested mind. As our Meeting Point for Early Mathematics Education is an institution that has been actively supporting children's interest in mathematics and the development of mathematical thinking for the last decades, we regard this progress with great pleasure. Furthermore, we are especially happy about the fact that the development of viewing mathematics as something challenging and enjoyable also is reflected in the statements of the ZMO-children participating in the activities of our Meeting Point (see the children's statements at the beginning and the end of this book).

As with any subject, interest in mathematics, mathematical competence and enjoyable activities in this regard can only arise when opportunities to access a variety of different experiences within the subject are created, preferably at a young age. Besides an early first, fundamental introduction, we consider it vital that especially talented children as well as children with special needs receive extra attention and care in order to build a strong foundation of mathematical knowledge and to encourage further development of skills. Especially the development of an orientation in the number space and a strong number construction sense has proven an important task in the early development of mathematical thinking (Schwank & Schwank, 2015). With regard to this issue, several Mathematical Playworlds allowing the introduction of mathematical concepts in a playful, action-oriented manner have been developed at our Meeting Point over the last years (Schwank, 2010 a, b, 2013, 2014). Concerning basic informatics education see also Dynamic Labyrinths (Schwank, 2016; see link below).

While our endeavors in this area find themselves embedded in a variety of efforts to support especially children with special educational needs (e.g. Kroesbergen & Van Luit, 2003; Storeygard, 2012; Kohli, Sullivan, Sadeh & Zopluoglu, 2015, Schwank 2013b), offers for children especially talented in the area of mathematics remain scarce.

In order to provide a contribution to filling this gap, we brought the Zingy Math Olympiad (ZMO) into being, an annual competition for mathematically talented 3rd graders, where they are given the opportunity to prove their abilities and solve challenging mathematical tasks in competition with other talented children. In the inaugural year, the Olympiad addressed the primary schools in the town of Osnabrück. Due to the great popularity of the first round in 2001, the radius was widened only one year later to include primary schools in the district, resulting in a catchment area consisting of approximately 120 primary schools. Together with their teacher, each participating class may choose one girl and one boy as class representatives to prove their talent in the Olympiad. From 2001 to 2013, a total of 2.102 children have participated, among these 1.063 boys and 1.039 girls. This small difference in numbers arose from the fact that especially during the first years, girls were considered less competitive by the sending schools so that only a boy but no girl from a class participated. From the beginning we were committed to ensure that boys and girls get the same opportunities which fortunately became more and more a matter of course for the schools and the participating children over the years (also see Chapter E. Further Perspectives). Based on their "Olympic performance" the children are sorted into four groups per gender (bronze, silver, gold, diamond [first three places]) and receive a certificate or – in the case of the first-placed boy and girl – one of the two ZMO challenge cups. In our view, this positive presentation of the children's mathematical competence and their enjoyment has always been a central part of this project.

The resulting wealth of specifically developed math problems and the individual solution approaches of the participating children created and collected over the course of 13 years builds the foundation of the present book. The intentions associated with this are manifold. Firstly, the collection of the Olympic tasks provides educational professionals with ideas on how to challenge especially talented children in their care. Secondly, the book includes several exemplary demonstrations of approaches the children chose in solving these tasks.

In a multi-value work (Schwank, 2016), an extensive selection of the most remarkable task processings with a wide spectrum of the children's ideas will be presented. Especially the reasons they gave for their processing allows a profound insight into children's strategies in mathematical thinking. Initially, this series will be published in German. Nonetheless it may be entertaining and instructive to follow the children's partly only little verbal mathematical undertakings.

Not only are these empirically diverse data of scientific interest, as so far, only little research on mathematically gifted children exists, but this work also offers practical hints on how to deal with various thinking processes children might use in a classroom or individual setting in order to facilitate targeted promotion.

A project of this size and nature be successful through the dedicated work of a great many people. We therefore heartily thank all who have been involved: first of all, the participating children and their teachers for their motivation and enthusiasm. Furthermore, the university students and employees as well as the volunteers for their tireless committed work. Finally, we want to express special gratitude to the Stiftung Stahlwerk Georgsmarienhütte whose financial support has been a substantial contribution to the ZMO's success.

We now wish the readers an enjoyable journey into the mathematical world of the ZMO. Our hope is to provide new insights into possibilities to positively influence children's understanding of mathematical concepts, structures and processes by opening opportunities for actively exploring the subject which last but not least also leads to chances to further enhance the increasing enthusiasm for mathematics.

Inge Schwank
for the Meeting Point »Early Mathematics Education«

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B. Collection of Olympic Tasks

In mathematics
the art of proposing a question
must be held of higher value than solving it.

~ Georg Cantor ~

Over a period of 13 years, the Meeting Point for Early Mathematics Education invited mathematically talented 3rd graders – one girl and one boy from each participating class – to take part in the Braintrain round of the Zingy Mathematical Olympiad (ZMO) and be challenged with exciting math problems, many of which they would probably not encounter during their ordinary math lessons in school.

Experiences from the first couple of ZMO rounds revealed that our selection criterion – the most mathematically talented girl and boy from each class – did indeed result in a sample of very high achieving students. This observation is consistent with recent findings indicating that teachers are rather successful, often even better than standardized measures, at detecting mathematically especially talented children among the students in their care (Niederer, Irwin, Irwin & Reilly, 2003; Hodge & Kemp, 2006). Therefore, our Meeting Point's team was presented with the challenge of developing Olympic tasks that on the one hand would be difficult enough to ensure the prevention of ceiling effects which is a problem that always has to be kept in mind when addressing high giftedness, no matter in which specific area (Thompson & Subotnik, 2010; Bortz & Döring, 2014). On the other hand, it was an important concern not to overextend the children who are still in the process of the development of their mathematical thinking with too demanding tasks, given that one important objective of the ZMO always has been to enhance children's joy in mathematics. Hence, we chose to intentionally include some tasks with a low item selectivity in order to make sure that all participating children would have the encouraging experience of successfully solving at least some tasks of an official math Olympiad, but also to incorporate sufficient tasks with a high item selectivity to be able to validly differentiate varying levels of mathematical talent among the participating children. .

Besides the item difficulty, we placed great emphasis on including a broad range of mathematical topics, not only from primary school math classes but also beyond that educational level to demonstrate the great diversity of possible math problems to the children and to allow us an insight into the processing strategies the children might use when dealing with a variety of mathematical topics. Tasks containing content exceeding the primary school math curriculum were included to examine the strategies children pursue when facing tasks for which they have not yet learned given approaches. On the one hand, this challenges the children to master a problem with their still limited knowledge about

mathematical formalism, e.g. usage of variables, terms and equations (understood in the sense of Krämer 2003, p. 171: formal mathematical writing as an intelligence amplifier), solely on the basis of their mental engagement in mathematical connections. On the other hand, several problems were created in a manner that allowed the children to first explore one or more given examples and then to generalize the discoveries made.

Finally, we would particularly like to emphasize that the tasks were not only designed to present a math problem and ask for the correct solution, but that almost every task sheet explicitly and deliberately requests the children to explain their solutions and give reasons for the approaches and strategies they used or chose not to use. This can be implemented by calculations, drawings or colloquial verbalization. Examples for important task elements are:

- "Some space for your thoughts and your answer."
- "Some space for your explanation. Calculate, draw or write something down."
- "Explain your answers ! Calculate, draw or write something down."

In our opinion, this reflection process is of prime importance not only to gain insight into the children's thoughts, but also to strengthen and consolidate the individual mathematical processing by consciously thinking about the mathematical operations and their functioning.

The result of our long-time and multi-facetted experience can be viewed in the following collection of 13 years of Olympic tasks. In order to grant a better overview, the tasks are sorted by topic instead of presenting the compilation of the individual annual rounds. Enjoy the read!

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Zingy-Math-Olympiad

First name, name _____

School _____

Class _____

Age _____

My math teacher's name (first name and surname) is

_____

Good luck
and success !

Table of Contents

Table of Contents (continued)

1 Attentive Calculation

Addition and Subtraction Facts

Name: _____

1 Solve the tasks !

$$\begin{array}{r} 384 \\ +\ 271 \\ \hline \end{array} \qquad \begin{array}{r} 906 \\ -\ 371 \\ \hline \end{array}$$

2 Oh dear, Crispin made a mistake !

$$\begin{array}{r} 473 \\ +\ 364 \\ \hline 109 \end{array}$$

Show Crispin how to do it right :

$$\begin{array}{r} 473 \\ +\ 364 \\ \hline \end{array}$$

What did Crispin do wrong ?

_____

_____

_____

3 Oh dear, Ann also made a mistake !

$$\begin{array}{r} 905 \\ -\ 286 \\ \hline 729 \end{array}$$

Show Ann how to do it right :

$$\begin{array}{r} 905 \\ -\ 286 \\ \hline \end{array}$$

What did Ann do wrong ?

_____

_____

_____

Name: _____

4 Solve the tasks !

$$\begin{array}{r} 285 \\ +\ 362 \\ \hline \end{array} \qquad \begin{array}{r} 807 \\ -\ 453 \\ \hline \end{array}$$

5 Oh dear, Cheng made a mistake !

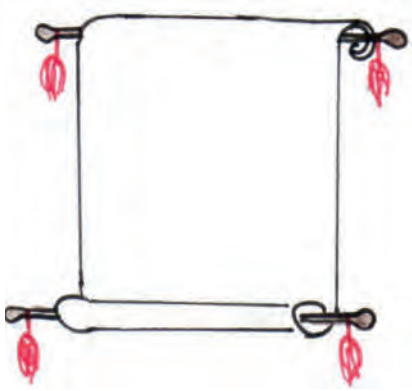
$$\begin{array}{r} 374 \\ +\ 265 \\ \hline 109 \end{array}$$

Show Cheng how to do it right :

$$\begin{array}{r} 374 \\ +\ 265 \\ \hline \end{array}$$

What did Cheng do wrong ?

Which other mistake can easily happen while calculating ?



Name: _____

6 **Did shark Sheldon pay attention in shark school ?**

He calculates:

$$\begin{array}{r} 2\ 8\ 7 \\ +\ 4\ 2\ 3 \\ \hline 6\ 0\ 0 \end{array}$$

Can Sheldon's result be right ?

Give Sheldon a tip on how to calculate better !
What should Sheldon bear in mind ?

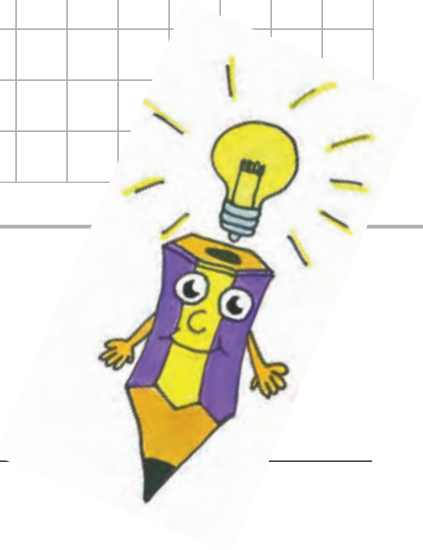
Name: _____

1 Calculate cleverly !

1 6 0 + 5 8 =

3 9 8 + 2 1 2 =

4 3 3 + 4 2 8 =



How can one calculate cleverly ?

_____

_____

_____

_____

_____

_____

_____

Name: _____

❷ Calculate especially clever !

1	2	7	+	3	9	8	=																

Calculate less clever !

1	2	7	+	3	9	8	=																

What are especially clever ways of calculating ?

_____

_____

_____

_____

_____

What are less clever ways of calculating ?

_____

_____

_____

_____

_____

Name: _____

1 Solve the tasks !

2 3 2	5 7 5	4 ▢ 3	9 2 ▢
+ 1 1 6	- 1 4 2	- ▢ 8 ▢	- 4 ▢ 1
_____	_____	_____	_____
		7 0 8	▢ 4 3

2 Try solving this task.

3 5 9 2 1 0 4 5 7
+ 2 7 1 6 5 3 8 8 2
_____

How could one process during calculating ? Why ?

_____

_____

_____

_____

3 Try solving this task.

3 2 0
- 5 2 0
_____

How could one process during calculating ? Why ?

_____

_____

_____

Name:

Even more space for your thoughts !

2 Number and Arithmetic Operation Riddles
Completing Calculations and Identifying Number Patterns

Name: _____

1 **What is missing ? Fill in.**

$26 + \square = 3 \times 10$

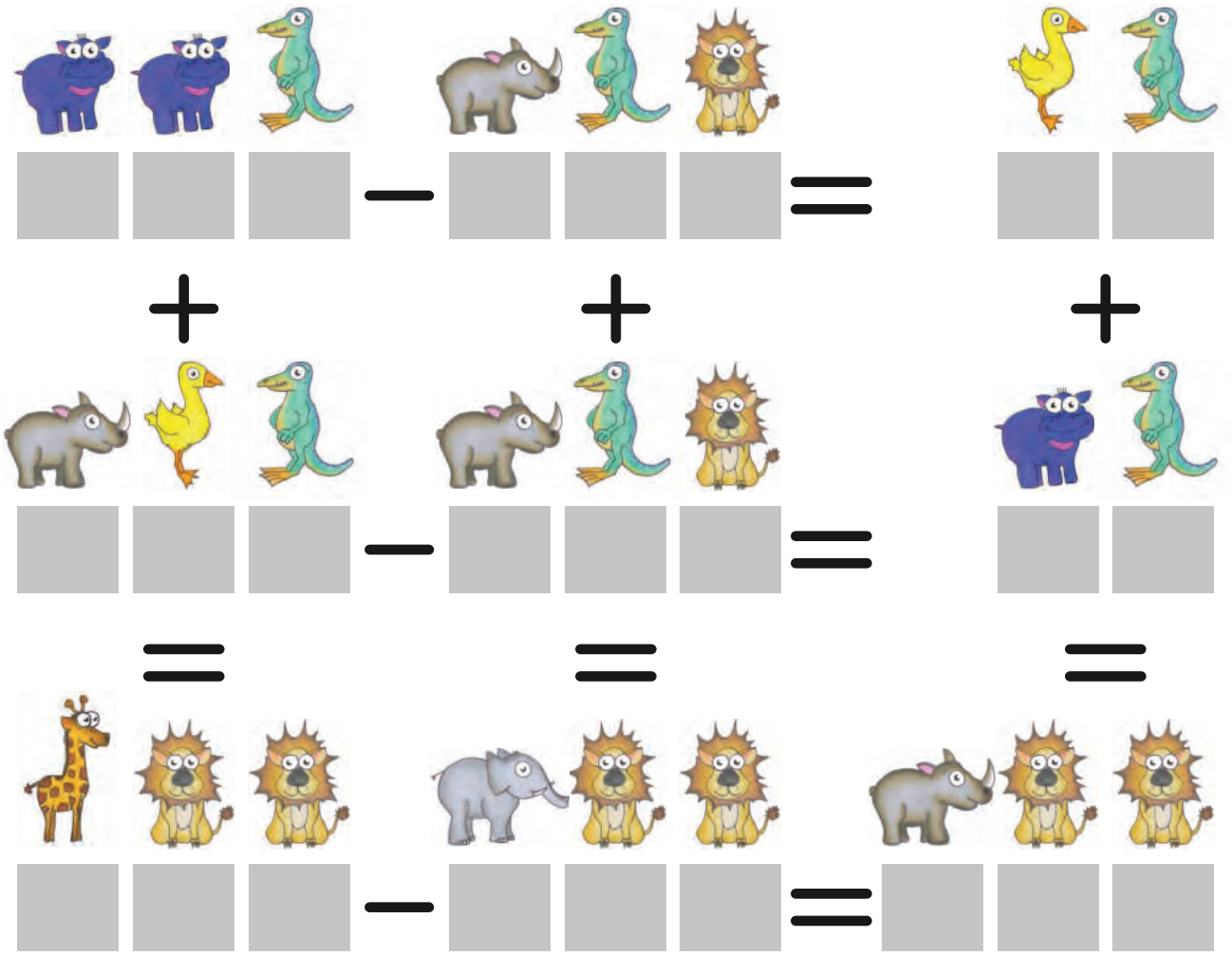
$497 + 8 - 8 + 8 - 8 + 3 = \square + 50$

$600 - 171 + 371 - 600 = \square + 120 + 358 - 358$

For calculating:

Name: _____

❷ In **Jungle School**, the **number sentences** are presented in a very special manner. Find out which digit is represented by each of the animal pictures.



Explain how you found the matching digits !

Name: _____

❸ **Find out which digit belongs to each letter !**

D	W	A	R	F

D = W − 156

R = F × 3

W = R + F

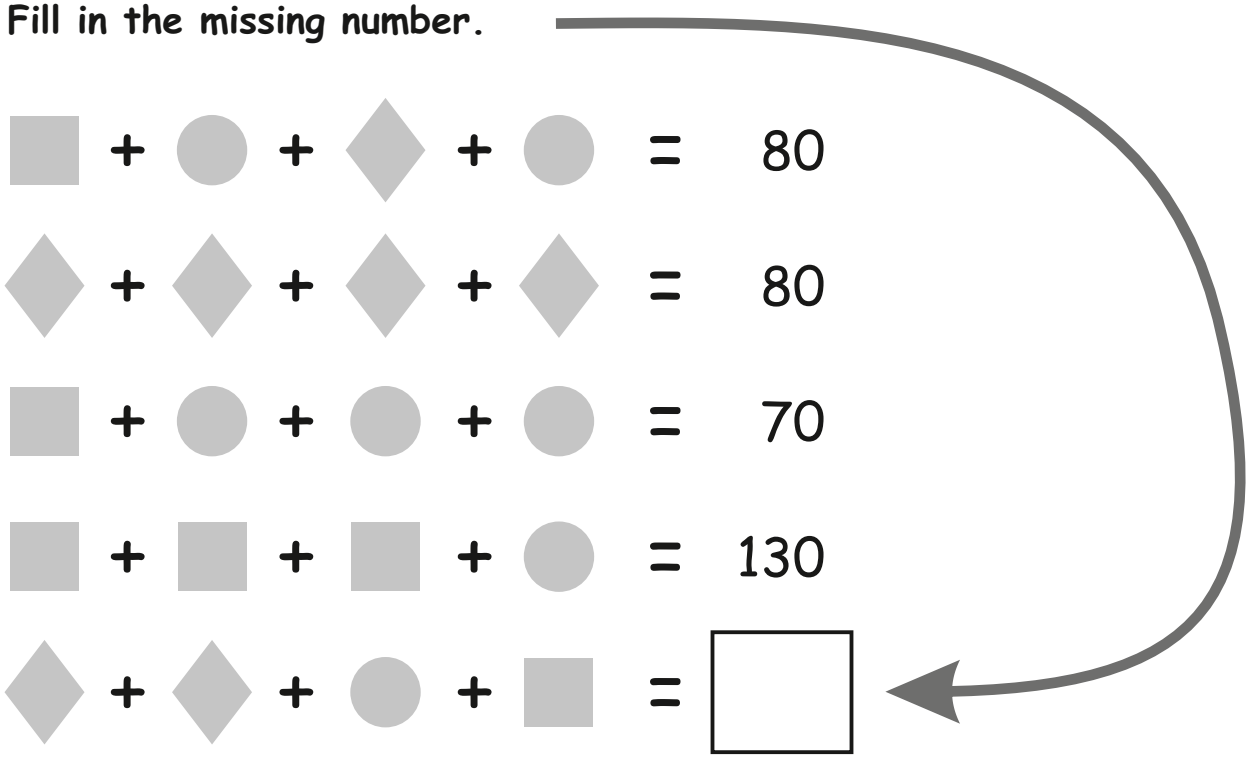
F = 328 − 286

D + W + A + R + F = 350

For calculating:

Name: _____

4 **Fill in the missing number.**



For calculating:

That's how I found my solution:

Name: _____

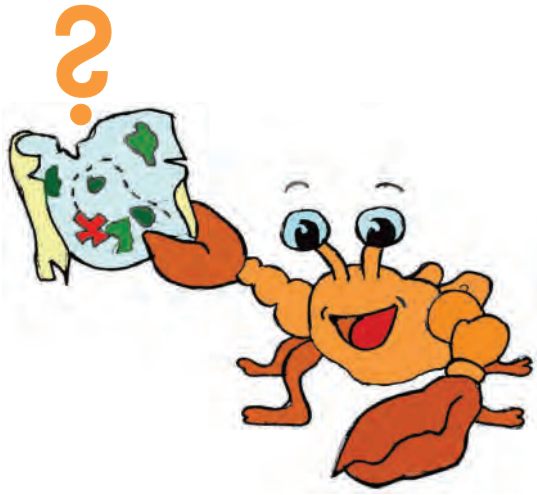
5 **Crabby has a treasure map.**

Here are some hints on how to reach the treasure:

♡ − ⬡ = 2

♡ : ⏢ = 2

⬡ + ♡ = 18

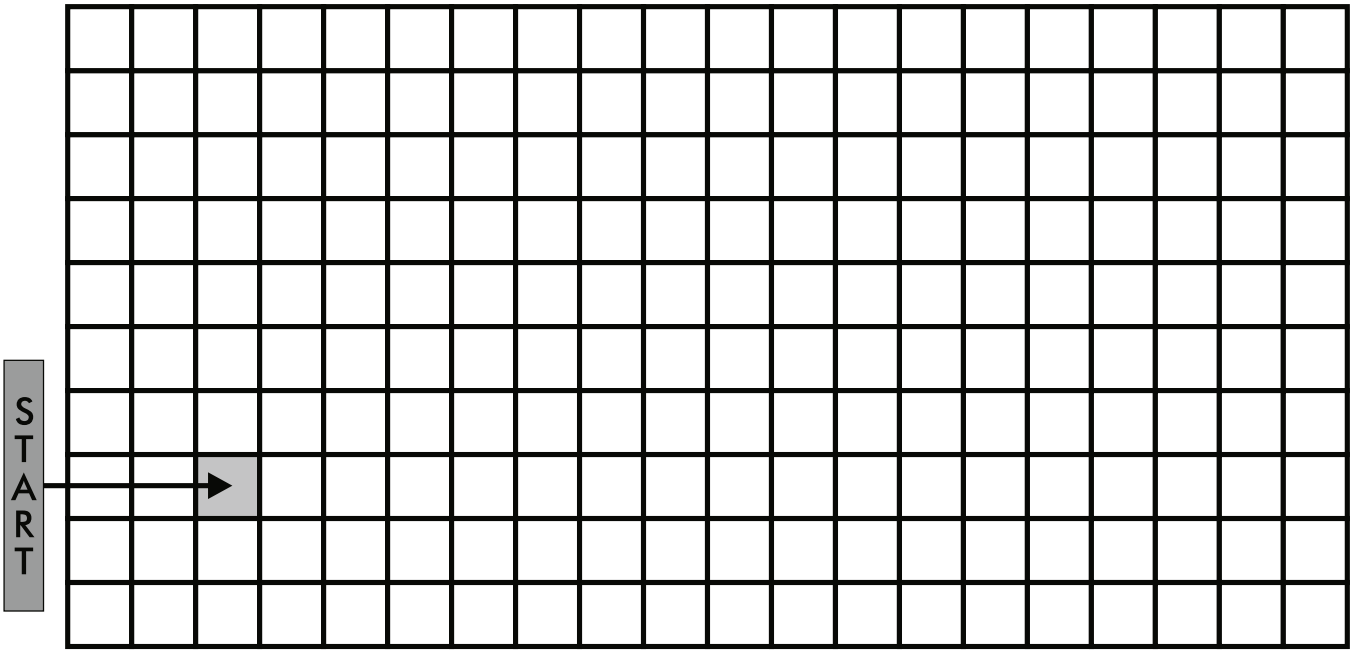


That's the way to the treasure:

From the starting point, walk ♡ steps to the right,

⏢ steps upwards and ⬡ steps to the left.

Draw the way to the treasure:



How did you figure out how to reach the treasure ?

_____

_____

_____

Name: _____

6 **Max is a puzzler.** He invented the following tasks:

● + 2 = ▲ ● · 2 = ◆

● − 2 = ■ ● : 2 = ■

Max discovered something. He found a digit for ● so that the sum of the results of his tasks above is 18.

▲ + ■ + ◆ + ■ = 18

Which digit did Max choose for ● ? _____

_____

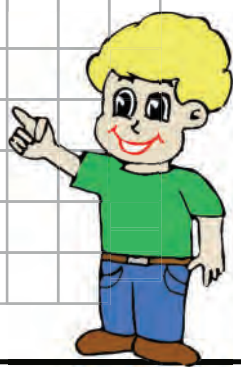
Trying out all possible digits isn't worthwhile, because:

_____

_____

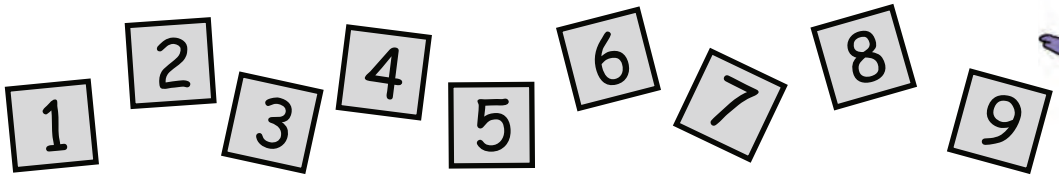
_____

For calculating:



Name: _____

7 **Roberta made these nine digit cards:**



Roberta wants to use her nine digit cards for putting them into a template for multiplication facts. Each digit may only be used once.

Which correct multiplication fact can Roberta put in here ?

□ • □ • □ = □ • □ • □

Give Roberta a hint on how she can find a solution.

_____

_____

_____

For calculating:

Could Roberta also use the card 0 for her task ?

_____

_____

_____

Name: _____

8 **Tiffany's tricky number task.**

Tiffany thought up two numbers.
Then she wrote down a calculation for these two numbers.

□ − ◇ = 2

Which numbers could Tiffany have thought of ?
Explain your answer !

Tiffany adds another calculation for her two numbers.
Both calculations have to be correct for her numbers.

Careful: before calculating the subtraction sentence in the second calculation, you first have to calculate the multiplication sentence.

□ − ◇ = 2

□ × □ − ◇ × ◇ = 20

Can you now tell exactly which numbers Tiffany thought of ?
Explain your answer !

Name: _____

1 **Lilly likes playing with numbers.** She loves calculations.
Today she ponders: " How can I get zero as
a result in a calculation ? "
Which ideas do you have ? Explain your ideas !

Lilly wants to accomplish the following calculation
by using only plus and minus.

Is that possible ? Give reasons for your answer !

Name: _____

❷ The multiplication- and division signs are missing.

Fill in × and : so that the calculations are correct.

100		10		10		10	=	10

72		9		5		4	=	160

For calculating:

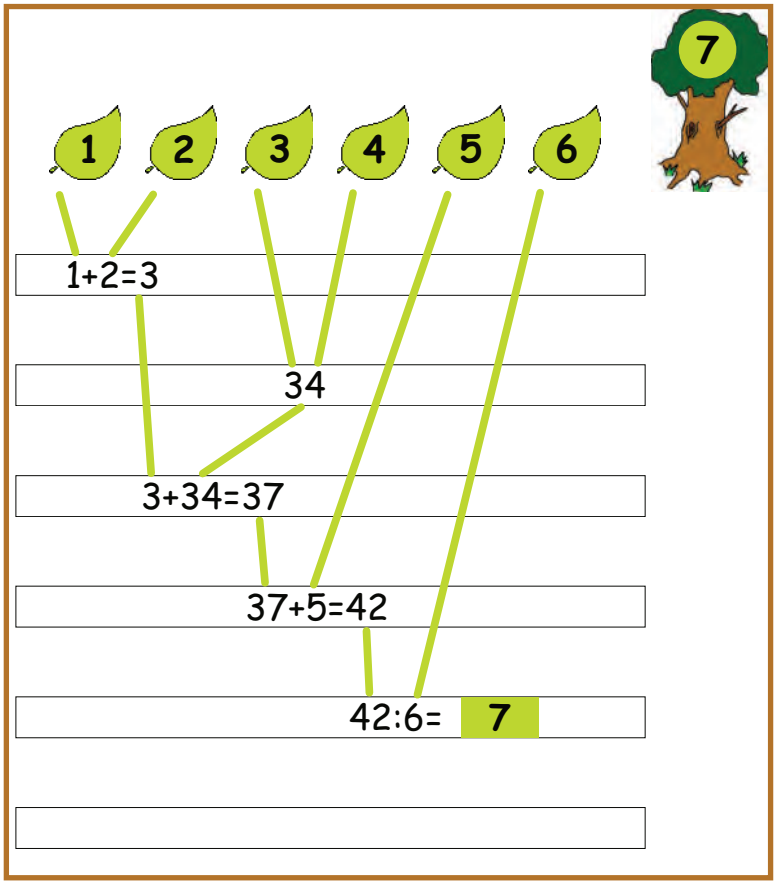
That's how I found my result:

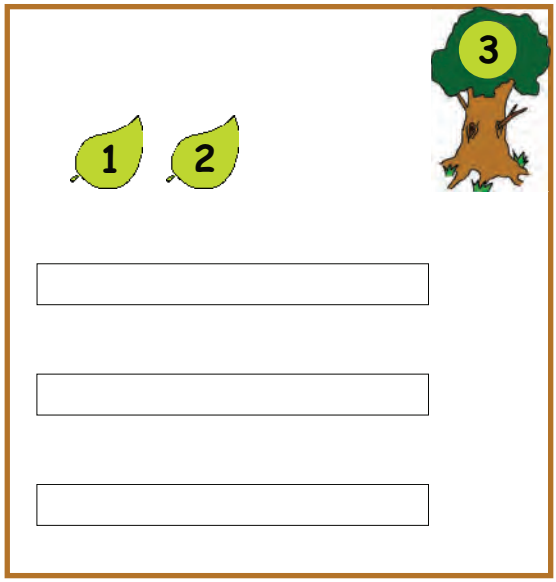
❸ **Eva likes creating math trees.** On this page you can see her solution for a tree with the end result **7** . She connects the leaves with calculation signs or uses them for a new number. She continues with this result until all leaves are connected with a branch and the correct end result is created.

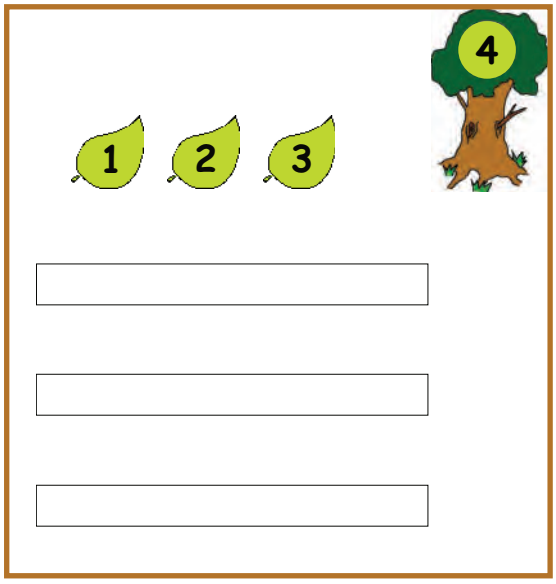
The branches may not cross and the order of the leaves may not be changed.

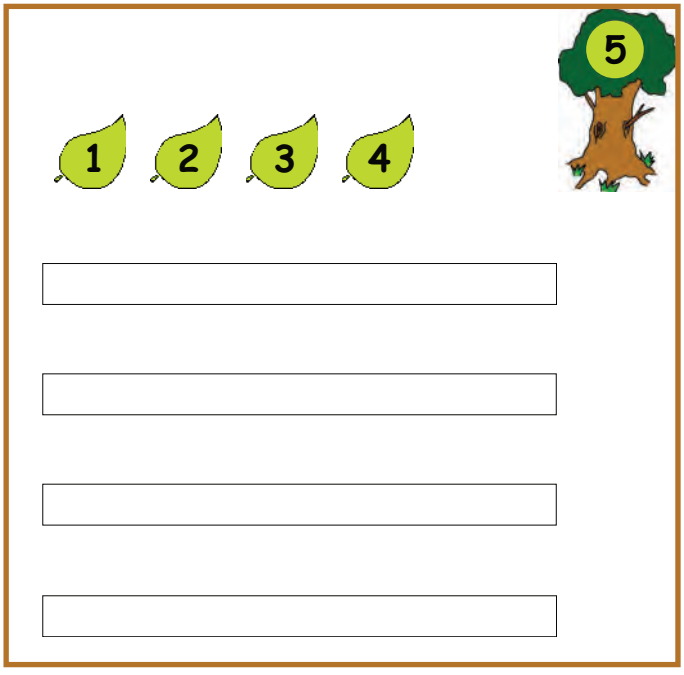
Try it out for yourself with the other trees.

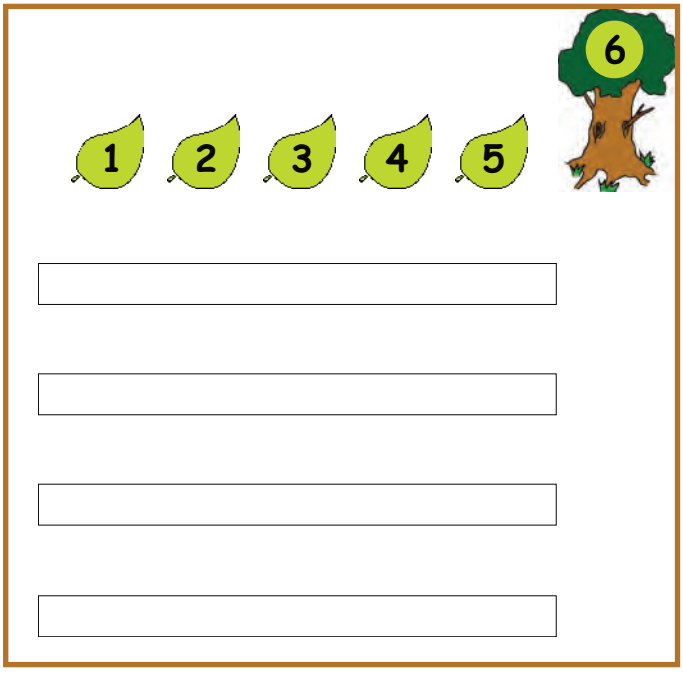
The tree with the end result **3** is very easy !





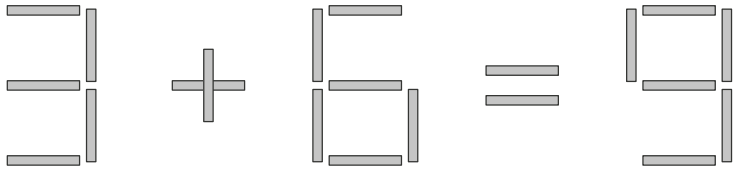






Name: _____

1 **Mary loves playing with matches.** She lays out the following calculation:



Mary notices: " If I change the position of just one match, I get a new correct result ! "

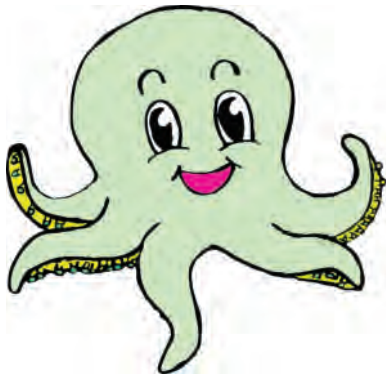
Draw Mary's new calculation.

Name: _____

1 **Kraky likes playing with numbers.**

Today she wrote numbers onto shells and
put these shells on the bottom of the sea.

2	3	5
5	1	4
3	6	1



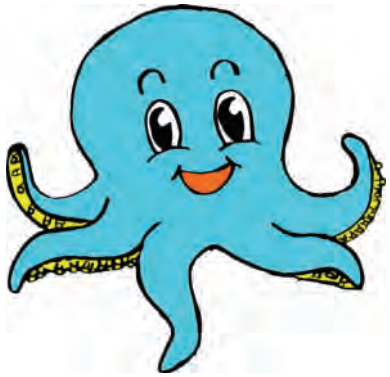
While putting the shells onto the sand, Kraky had something in mind. What might that have been ? Which rule might she have used ?

_____

_____

Kraky's friend also lays out a pattern.

1	2	6
5	4	0
3	3	3



He states: "My rule fits both patterns."
Is Kraki's friend right ? Is there a rule that fits both patterns ?

_____

_____

_____

_____

Name: _____

❷ **Alex, Ben, Chris, Dan und Elias live in Number Ally.**

This week they want to decorate their houses with numbers.

Alex, Ben and Chris are already finished.

Fill in the numbers for Dan and Elias.

Alex	Ben	Chris	Dan	Elias
1	6	2		
4 | 3	7 | 1	8 | 6	14 | 2	|
7	8	14		

Which rule might the boys have used for their number decoration ?
Several rules are possible !

Name:

Even more space for your thoughts !

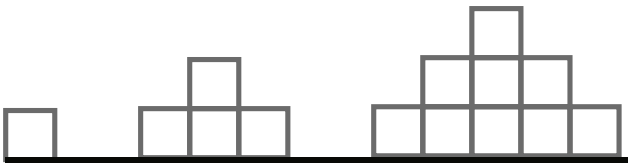
3 Arithmetic Regularities

Identifying Arithmetic Regularities in Numerical Series and Specified Number Sentences

Name: _____

1 **Princess Mia likes playing and calculating with pads.**

Today she lays out a row of pads like this:



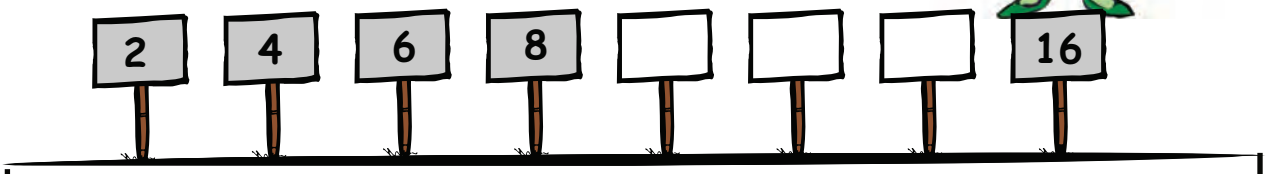
How can the row be continued ?

What could Mia calculate ?

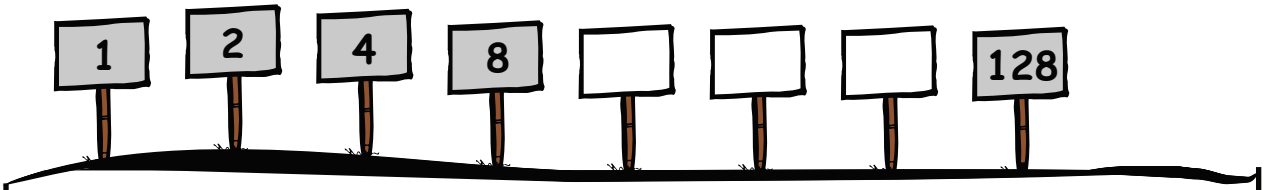
Name: _____

❷ **The wicked witch of Fairyland cursed away some numbers.**

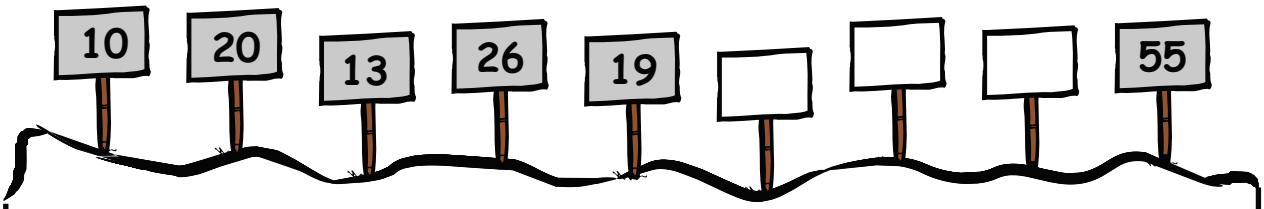
Help the inhabitants of Fairyland to rebuild their pretty number sentences !



Think carefully and write numbers in the empty signs.
Why are your numbers fitting ?



Think carefully and write numbers in the empty signs.
Why are your numbers fitting ?



Think carefully and write numbers in the empty signs.
Why are your numbers fitting ?

Name: _____

❸ The castle children like numbers.

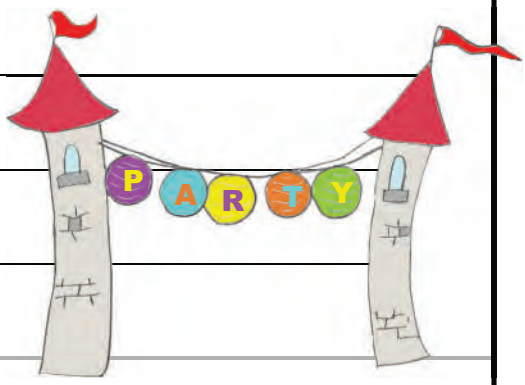
For their party they create garlands with numbers.
Continue the row on the first garland.

Finding the next numbers is easy, because:

_____

_____

_____



Continue the row on the second garland:

Finding the next numbers is easy, because:

_____

_____

_____

Continue the row on the third garland:

Finding the next numbers is easy, because:

_____

_____

_____

❹ **Lisa likes numbers.** She enjoys writing them in rows. Continue !

Lisa's first row:

4, 8, 12, 16, ___, ___, ___

Lisa thinks: I can easily calculate the next numbers, because:

_____

_____

Lisa's second row:

2, 4, 8, 16, ___, ___, ___

Lisa thinks: Again, I can easily calculate the next numbers, because:

_____

_____

Lisa's third row:

1, 4, 9, 16, ___, ___, ___

Lisa thinks: Again, I can easily calculate the next numbers, because:

_____

_____

Lisa ponders: 4 and 16 appear in all three rows. Are there other numbers appearing in all three rows ? Which numbers are these ?

_____

_____

_____

_____

_____

Name: _____

1 **Lara thought up some number sentences.** Solve them.

2	+	3	+	4	=											
4	+	5	+	6	=											
6	+	7	+	8	=											
2	1	+	2	2	+	2	3	=								

Lara thought up very special number sentences.
How did she choose her numbers ?

Lara notices that the results of her number sentences are very special numbers. What is special about her result numbers ?

Why do the result numbers become so special? Does this work with many or even a lot of these number sentences ?

❷ **Pam wants to calculate special addition facts.**
She starts by writing **42** as her first number.
Then she switches the digits and writes: **24**.
She adds these two numbers: **42 + 24**.
As a result she gets:

Once again, Pam writes a number: **53**.
Then she switches the digits and writes: **35**.
And again she adds the two numbers: **53 + 35**.
As a result she gets:

What is remarkable about Pam's results ?

Pam ponders: If I start with any other number from **10 to 99**, will I always get a special result with my **addition fact** ?
What do you think ?

Some space for your thoughts and your answer:

Name: _____

1

Calculate:

$2 \cdot 3 =$ $1 \cdot 4 =$

$3 \cdot 4 =$ $2 \cdot 5 =$

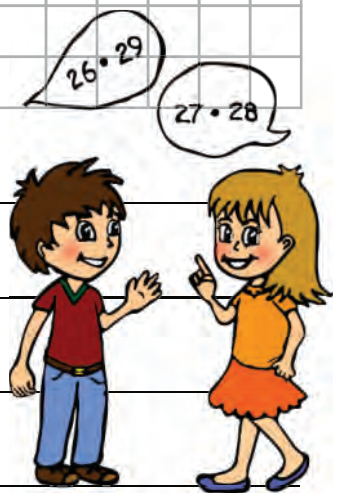
$4 \cdot 5 =$ $3 \cdot 6 =$

Continue:

What do you notice ?

Ivy knows: The result of $27 \cdot 28$ is 756.
Finley is happy. He has to calculate the result of $26 \cdot 29$.

What does Finley calculate ? Why is he happy ?



❷ **Marty's marvellous math trick.** Marty's favorite numbers are:
4, 9, 16, 25, 36, ... What is remarkable about Marty's favourite numbers ?

_____

_____

Write down three more of Marty's favourite numbers:

Marty often wants to use his favorite numbers. When he is supposed to calculate 3×5 , he calculates 4×4 instead and subtracts 1 from the result. Calculate like Marty and continue filling out the table !

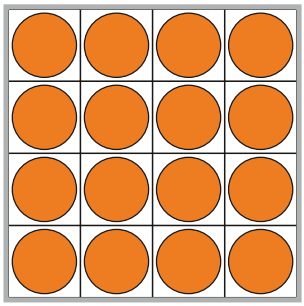
Task	Marty's way of calculating		Result
3 × 5	4 × 4 = 16	16 - 1 =	3 × 5 =
4 × 6	5 × 5 =	- 1 =	4 × 6 =
7 × 9	× =	- 1 =	7 × 9 =

Why do Marty's calculations always reach the correct result ? You can demonstrate your ideas in many ways: on the numberline or calculation strip or with the help of the array model. Do it as you like !

Some space for your thoughts and your answer:

Name: _____

❸ **Cato especially likes calculations with patterns.**
He thinks this pattern is great:



Cato explains: "The pattern helps me. In the pattern I can see that 4×4 is the same as $3 \times 5 + 1$."
Does the pattern fit to Cato's calculation ? Explain !

_____

_____

_____

Cato is proud : "With my pattern trick I can very well use neighboring numbers while calculating."
Write the number sentences with the neighboring numbers just like Cato.

$3 \times 3 =$ _____

$5 \times 5 =$ _____

$6 \times 6 =$ _____

Can Cato use his pattern trick for all multiplication sentences with two identical numbers and their neighboring numbers ?
Explain ! You can also draw something.

4 Operating with Options

Getting an Overview over Different Possibilities, Determining their Quantity or Choosing a Suitable Option

Name: _____

1 **Before starting to paint, painter Patty likes to prepare plans.**

For a new wall she thought up a pattern with the colors black, red, yellow.

Finish her pattern.

b	y	
r	b	
y	r	

b: black

r: red

y: yellow

Why is the way you finished the pattern fitting ?

_____

_____

_____

Patty invents another pattern with a fourth color.

b	g		
r	b		
y	r		
g	y		

b: black **g**: green

r: red

y: yellow

Patty wonders which possibilities she might have to continue the pattern.

Give her a clue ! Which color would fit into the field the arrow is pointing ?

Explain your answer !

_____

_____

_____

_____

Name: _____

❷ **Today, farmer Colin weighs his cows.** He notes:

Lucy is lighter than Erica.
Harriet weighs 5 kg more than Gloria.
Erica weighs 3 kg more than Gloria.
Gloria weighs less than Lucy.

Arrange the cows according to their weight.
Write or draw your result.

Which cow is the lightest ? _____

Which cow is the heaviest ? _____

❸ **Frank, Owen, Jacob, Tina, Eva and Lily are friends.** Today, they are riding on the bus together. Luckily they found three double seats for themselves that are directly behind one another.

Frank is sitting in front of Tina and behind Owen.
Jacob is sitting next to Tina.
Lily is sitting next to Owen and in front of Jacob.

Find out how the children might be sitting next to and behind one another on the bus. If you like you can create name cards for solving the task and put them on the fields for the seats. Afterwards, write down your solution.

For placing

Seat	Seat
Seat	Seat
Seat	Seat

For writing

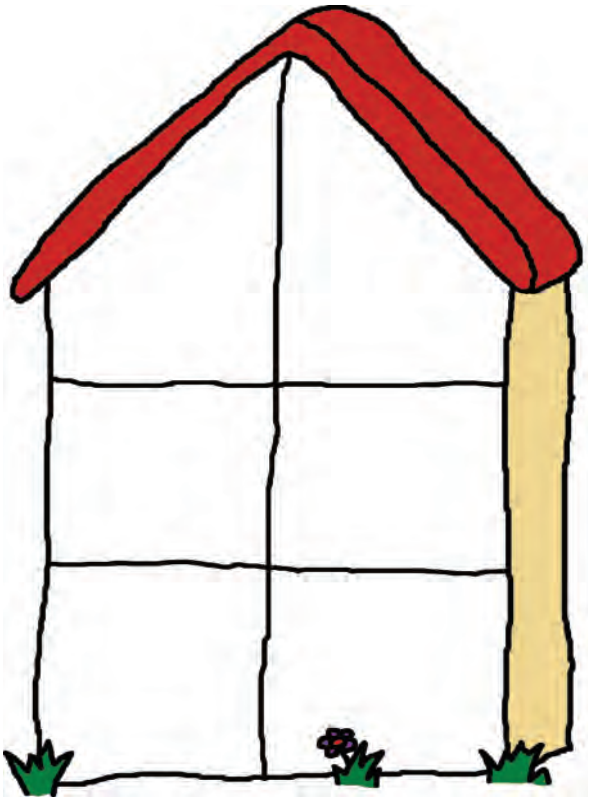
Seat	Seat
Seat	Seat
Seat	Seat

Describe who sits next to and behind Eva.
Explain your answers.

Name: _____

4 **Once again, postmaster Casper delivers letters to the monastery.**

That's what he remembers about the inhabit
Joseph lives on the left of Simon.
Aaron lives on the right of Freddy.
Joseph lives on a higher floor than Freddy.
Simon lives on a lower floor than Martin.
Neither Martin nor Joseph live on the right side.
Sadly he doesn't remember anything concerning Brian.



Help the postmaster and fill in the names at the correct location in the house !

Where does Brian live ? Explain !

Name: _____

5 **Fairy Fiona is having a party with her friends. All of them are sitting at a round table.**

Fiona herself is sitting between Becky and Annie. Lauri is sitting opposite of her. Grace is sitting on the left of Lauri. Annie is sitting opposite of Grace. There is one seat left for Maisie. Where is she sitting ?

_____

_____

Some space for your explanation.
Calculate, draw or write something down.

Name: _____

6 **Little Merry owns three hats:**

A felt hat, a straw hat and a leather hat.
One hat is red, one hat is blue, one hat is green.
One hat has one feather, one hat has two feathers and one hat has three feathers.

This is his hat riddle for you:

- My hat with one feather isn't green.
- My felt hat isn't red and has one less feather than my leather hat.
- My blue hat has three feathers.
- My leather hat doesn't have two feathers.

Find out the color of each hat.
Find out the number of feathers each hat is decorated with.

Explain your answers ! Calculate, draw or write something down.

Name: _____

1 **Benjamin wants to buy himself some icecream for 1,71 €.** There are various coins in his wallet:

three 1 € coins,
three 50 cent coins,
three 20 cent coins,
three 10 cent coins,
three 5 cent coins,
three 2 cent coins.

Name five **different** options for Benjamin to pay for his icecream with the exactly right amount of money. Always using the same coins in just a different order does not count.

1st option: _____

_____

2nd option: _____

_____

3rd option: _____

_____

4th option: _____

_____

5th option: _____

_____

What was your approach in the search for the solutions ?

_____

_____

_____

_____

Name: _____

1 **Peter is bored.** He throws two dice and adds up the points.

The first time he throws  . This adds up to 8 points.

What is the lowest amount of points one can get with 2 dice ?

What is the highest amount of points one can get with 2 dice ?

Fill out the table with all possible amounts of points:

						
	2	3				
	3					
						
						
						
						

Peter continues throwing dice a very long time. All in all he rolls them more than 1000 times ! Which amount of points did he probably reach most often ? Explain your answer !

Name: _____

❷ **In the afternoon, there is a Dog Show with Ari, Bla and Cos.**
In the beginning, each of them is sitting on one of three different boxes.

During the show their task is to change their seats again and again in new ways. Of course, this only works for some time.
Which different options are there for Ari, Bla and Cos to choose a box ?

This is how Ari, Bla and Cos can sit:

All in all, Ari, Bla and Cos have ____ **different** options.

What do you have to keep in mind in order to find all options ?

Name: _____

❸ **Kim und Ken invent new words.** These words all have 4 letters and are written only with the letters **a**, **e**, **n** and **g**.
The last letter always is a **n** or a **g** and every letter appears only exactly once in every word.
Kim starts and writes: **aeng**.
Ken says: "We'll be able to invent lots and lots of words! Surely more than 100."

Find out, how many words there really are !
Write down all possible words.

There are _____ possible words.

Explain why it's not possible for Kim and Ken to invent a lot of words using their rules.

Name: _____

❹ **Mermaid Nixi has a wardrobe full of ribbons.**
She owns ribbons in five different colors:
red, blue, green, orange and purple.

Every day she wears three ribbons
in three different colors.

How many different outfits can she compile ?

Explain your answer ! Calculate, draw or write something down.

Name: _____

5 **Dwarf Baku wonders**
what he can wear to the party.
He has four different sweaters,
one pair of red, one pair of blue and
one pair of yellow trousers
as well as two different pointed hats.

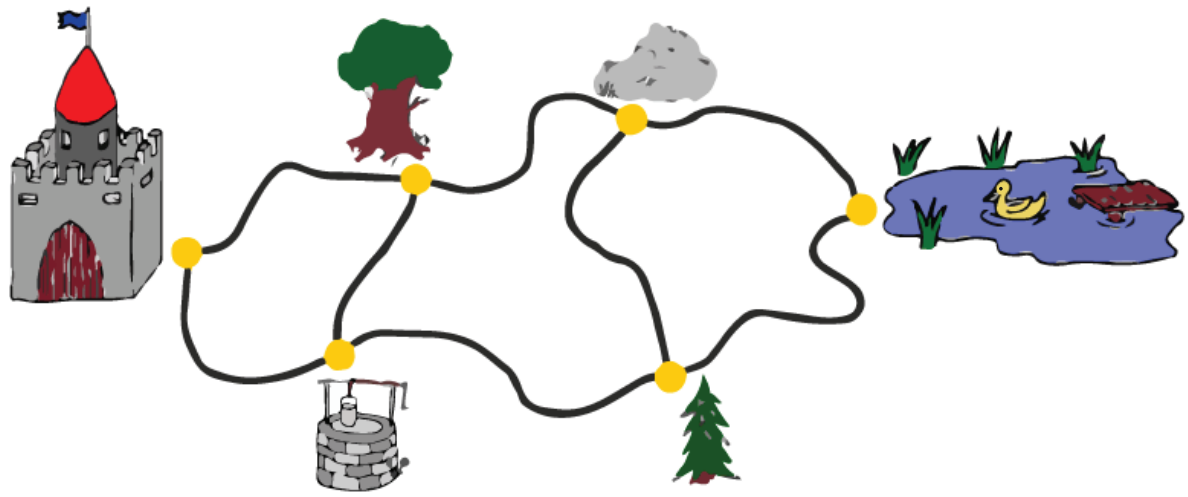
Dwarf Baku definitely wants to wear one sweater,
one pair of trousers and one pointed hat.
For this he has many options.
How many are there exactly ?

Some space for your explanation.
Calculate, draw or write something down.

Name: _____

6 **Prince Charming likes to ride different paths.**

He has several options to get from the castle to the lake.



Prince Charming wonders:

How many options do I have to ride from the castle to the lake ?

I don't want to use any section of the path twice during my ride.

Prince Charming has _____ options.

Explain your answer !

You can use the drawing with the paths for your explanation.

5 Word Problems

Mastering Problems Formulated in Text Form with a Mathematical Perspective

Name: _____

1 **Penny and Tom are collecting shells.**
Penny finds 26 shells, Tom only 12.

How many shells does Penny have to give to Tom so that both have the same amount of shells ? _____

How many shells does Tom now have ? _____

Think and draw:

Or calculate:

Name: _____

❷ **Every day, Lizzy feeds her Bambi the same amount of cookies.**
Over a period of five days, Lizzy needs 30 cookies.
How many cookies does she feed her Bambi in two days ?

Answer: _____

_____

Explain your solution !
you can calculate, draw or write.

Name: _____

❸ **Upon a large jungle tree, many birds are sitting together with their leader Logi.** On a neighboring tree the two chimpanzees Pa and Pu are sitting. Pa calls out to the birds: “Hello you 200 birds!” Logi answers: “We are not that many. But if you add to us the double of our number and also count the two of you, then we would be 200 on this tree.”

How many birds are sitting on the tree ? _____

Some space for your thoughts.
Calculate, draw or write something down.

1 How old are the two goblins?

Together we're 40 years old.

I'm 30 years older than you !

Age: _____ years

Age: _____ years

Explain your answer !
You can calculate, draw and write.

Name: _____

❷ **Tony owns sheep and pigs.** His sister Mary also owns sheep and pigs. Together they own 30 animals. Frank is coming over and says "That's interesting. Mary owns as many animals as Tony's animals have legs."

How many animals does Tony own ? _____

How many animaly does Mary own ? _____

Some space for your thoughts.
Calculate, draw or write something down.

Name: _____

❸ **A farmer sells five sheep to her neighbor.**

For each brown sheep she gets 105 Euro.

For each white sheep she even gets 15 Euro more.

Her neighbor gives her 570 Euro for the five sheep.

How many of the sold sheep are white ?

_____

Some space for your explanation.
Calculate, draw or write something down.

Name: _____

4 **There are 32 bedrooms and a total of 57 beds in the hotel "School of Fish".**

There are either two beds or just one bed in the bedrooms.

How many bedrooms with one bed can there be ? _____

How many bedrooms with two beds can there be ? _____

Explain your answer ! Calculate, draw or write something down.

A bedroom with one bed costs 25 € per night.
A bedroom with two beds costs 30 € per night.
The Fissy family and their friends pay 305 € for an overnight stay.

How many rooms with one bed can they have booked ? _____

How many rooms with two beds can they have booked ? _____

Explain your answer ! Calculate, draw or write something down.

Name: _____

5 **Joali wants to buy coconuts at the market.**
The trader enjoys calculating. She put up two price signs stating:
6 light and 8 dark coconuts cost 18 €.
9 light and 4 dark coconuts cost 15 €.
Joali has only little money with her.
She can only buy 1 light and 1 dark coconut.
How much does she have to pay ?

Joali has to pay _____ €.

Some space for your thoughts.
Calculate, draw or write something down.

Name: _____

6 **At the festival, Fairy Nora** buys two fried sausages and six cups of apple juice for herself and her friends.

All in all she pays 12,60 Euro.
Two fried sausages cost exactly as much as three cups of apple juice.

How much is one fried sausage ? _____

How much is one cup of apple juice ? _____

Some space for your explanation.
Calculate, draw or write something down.

Name: _____

1

A rabbit in a box weighs 4 kg.
A duck in the same box weighs 5 kg.
Together, duck and rabbit weigh 3 kg.
How heavy is the box ?

Answer: _____

Some space for your explanation.
Calculate, draw or write something down.

Name: _____

❷ **Castle girl Ivy loves long hiking tours.**

Last week, she covered a total distance of 48 km over three days.
On Wednesday she hiked twice as far as on Monday.
On Friday she hiked three times as far as on Monday.
Which distance did she cover at the respective days ?

Monday: _____

Wednesday: _____

Friday: _____

Explain your answers ! Calculate, draw or write something down.

Name: _____

❸ **Uta describes:**

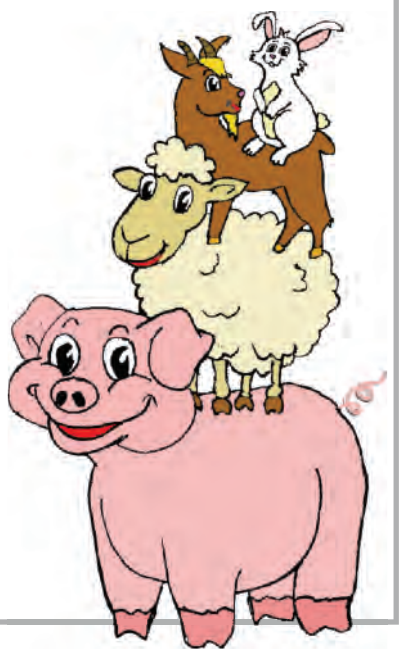
"There are four different animal species at my holiday farm.
All in all there are 104 animals. The number of the sheep and the pigs is the same. There are two more rabbits than goats. And there is one less pig than there are goats."

How many sheep, pigs, rabbits and goats does Uta own ?

_____

_____

Some space for your explanation.
Calculate, draw or write something down.



Name: _____

4 **Today, the cook is buying fruit:** One sack of apples, one sack of pears and one sack of oranges.
One sack of apples costs just as much as one sack of pears.
One sack of oranges costs 10 Thaler more than two sacks of apples. In total she has to pay 26 Thaler.
How much is one sack of oranges ?

For one sack of oranges she has to pay _____ .

Explain your answers ! Calculate, draw or write something down.

❺ **Circus Puzzolino currently has 224 guests.** There are 38 more adults than boys and 6 more boys than girls. How many girls, boys and adults are in the audience ?

Girls: _____

Boys: _____

Adults: _____

Think and try:

For calculating:

Name: _____

1 **A piece of land is getting a new fence.**
The land plot is 84 m in length and 60 m in width. Some space has to be left free for two gates. One gate is 4 m wide.

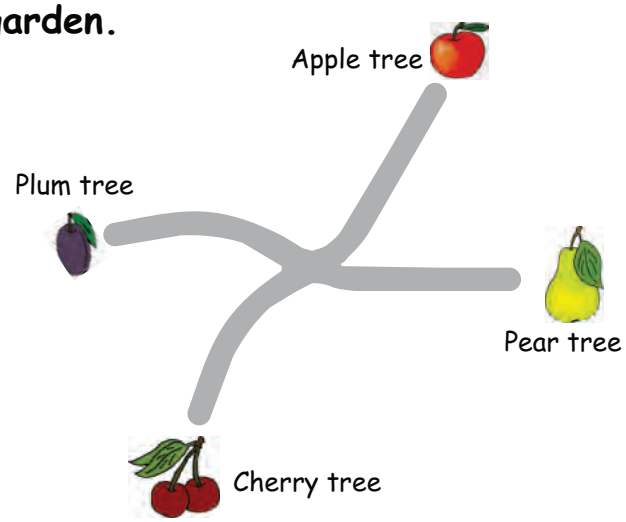
Material for how many metres of fence is needed?

_____

Some space for your explanation.
Caluclate, draw or write something down.

Name: _____

❷ **Grandma Laurel planted 4 trees in her garden.**
In between there are some paths.
Every evening she goes for a stroll on these paths. She walks from the apple tree to the pear tree, that's 10 m. She continues to walk from the pear tree to the cherry tree, that's 6 m. She then proceeds from the cherry tree to the plum tree, that's 14 m. Afterwards, she walks from the plum tree to the apple tree.



Grandma Laurel thinks: "The walk from the plum tree to the apple tree is longer than the walk from the pear tree to the cherry tree." Is she right ?
Explain ! _____

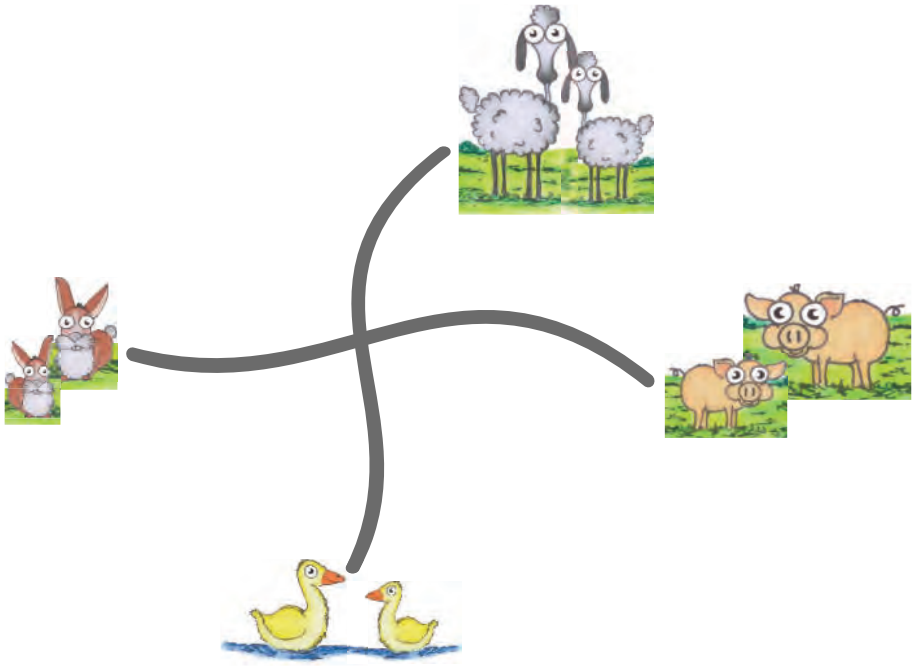
_____

_____

When walking directly from the plum tree to the apple tree, how many metres does Grandma Laurel have to walk ?
Calculate.

Name: _____

❸ **The twins Tina and Sina are training for the sports festival.**
They made a sketch of their route.



Tina suggests: "We start at the rabbits and run the 120 m to the sheep. Then we run the 140 m to the pigs. Afterwards we run the 100 m to the ducks and finally back to the rabbits."
Sina agrees: "Excellent, then the last section is the shortest."

Is Sina right ? How long is the route from the ducks to the rabbits ?
Explain your answer ! You can also use the sketch.

Name: _____

❹ **Michael and Pansy go to the same school.** Michael lives 3 km away from the school, Pansy 4 km. After lunch Pansy wants to cycle to Michael.
How far might the distance be ?
There are several possibilities.

Think carefully and draw:

That's how I found my result:

Name: _____

5 **Sue and Larry are friends.** They go to the same primary school. Sue lives 2 km away from school, Larry 3 km. In the afternoon, they often meet. Today, Larry wants to cycle from his place to Sue's place. He chooses the shortest route. How long might his way be ?

Careful: there are several possibilities.

Some space for your thoughts and your answer:

Why are there so many possibilities how long Larry's shortest route might be ?

Name: _____

1 **Sea horse Turbo-Fred wants to visit merman Max.**

Right now it is 10 o'clock and Turbo-Fred still has 20 m to swim.

He thinks: "I need 14 minutes to swim 2 m.
I won't be having any more breaks.
Therefore I will arrive at _____ o'clock."

Explain your answer !

Calculate, draw or write something down.

In fact, he meets Max earlier, because Max starts out at 10 o'clock to meet him on the way. Max can swim 3 m in 14 minutes.
When do the two meet ? _____

Explain your answer ! Calculate, draw or write something down.

Name: _____

❷ **The dwarves Gevi, Gefu und Gese want to wrap lots of presents for their party.**

Gevi manages to wrap four presents per hour.

Gefu manages to wrap five presents per hour.

Gese manages to wrap six presents per hour.

Today, Gevi secretly got up earlier. He has already wrapped eight presents when Gefu and Gese also start wrapping.

Gefu brags: "In two hours I'll catch up with Gevi !"

Gese contradicts: "No, I'll catch up with Gevi in two hours !"

What do you think ?

When will Gefu catch up with Gevi, when will Gese ?

_____

_____

Some space for your explanation.
Calculate, draw or write something down.

Name: _____

❸ **Ali, Bea und Cero are making plates out of clay.**
Ali can produce 4 plates per hour.
Bea can produce 5 plates per hour.
Cero can produce 6 plates per hour.
Today, Ali secretly got up earlier.
He has already produced 8 plates when Bea and Cero also start their work.

Bea brags: "In two hours, I'll catch up with you !"
Cero contradicts: "This cannot be ! Anyhow, I'll catch up with Ali first."

Who is right ? Exactly how many hours will it take Bea and Cero to catch up with Ali ?

Some space for your thoughts.
Calculate, draw or write something down.

Name: _____

1 **Getting up is difficult!** In their dormitory, the knights use the following method: All the knights that are already awake go round to wake their still sleeping comrades. They are very fast. Waking a knight takes only one minute.

Today, all the knights are still asleep as rooster Harry starts crowing. Only the youngest knight wakes up.
It is 7:00 o'clock.
How many knights are awake at 7:01 o'clock ? Explain !

How many knights are awake at 7:02 o'clock ? Explain !

How many knights are awake at 7:03 o'clock ? Explain !

At 7:10 o'clock all knights are awake. How many are they ? Explain !

Name: _____

1 **Today, the 23 children of class 3b make a trip to the ice rink together with their teacher Ms. Clever.**

The entrance fee is 3 Euro per person.
Ms. Clever already collected 64 Euro and is puzzled. Why is she puzzled ?

_____

_____

_____

_____

How much is the total entrance fee ? _____

Explain your answer ! Calculate, draw or write something down.

Name: _____

❷ **It is Sunday.** Pitch Mary, Golden Mary and Mother Hulda meet at the well.

In how many days will all three meet again at the well ?

_____

_____

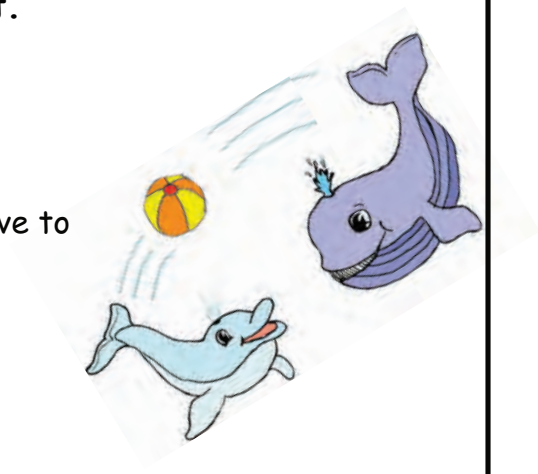
Explain your answer !
You can calculate, draw or write something down.

Name: _____

1 **This year, ten teams compete in the Finball Contest.**

Each team meets every other team twice.

There are Gold and Rust points.



That's how the points are accounted when a team loses:

- In case a team still has Gold points and loses, they have to surrender one Gold point.
- In case a team has no Gold points and loses, they get one Rust point.

That's how the points are accounted when a team wins:

- In case a team still has Rust points and wins, they may get rid of one Rust point.
- In case a team has no Rust points and wins, they get one Gold point.

In case of a draw, the score doesn't change for either team.

In the first two matches of the year, team Dolphy competes againts team Whala.
Team Dolphy wins both matches.
What is the current score ? Explain !

_____

_____

Team Octo reached the end of the playing time.
In how many matches did they participate ? Explain !

_____

_____

By the end of the playing time, team Octo managed to get three Gold points. How many times might team Octo have won ? How many times might they have lost ? How many of their matches might have ended with a draw ?
Explain !

_____

_____

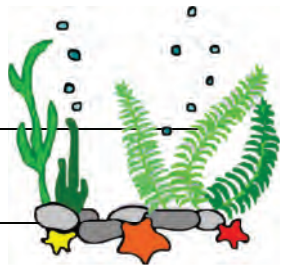
_____

_____

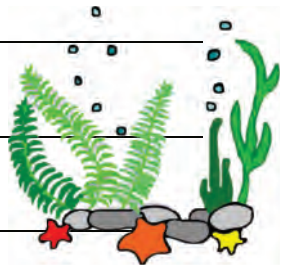
Name: _____

❷ **For his birthday, Granny gives Barney a fish tank with wonderful fish.**
There are 6 red fish, 3 blue fish and 9 green fish.
The fish are so special because they can change their color.

When a red fish touches a green fish, both turn blue.
When a red fish touches a blue fish, both turn green.
Which rule do you recognize ? Explain your rule !



When a red fish touches a red fish, both stay red.
When a green fish touches a green fish, both stay green.
Which rule do you recognize ? Explain your rule !



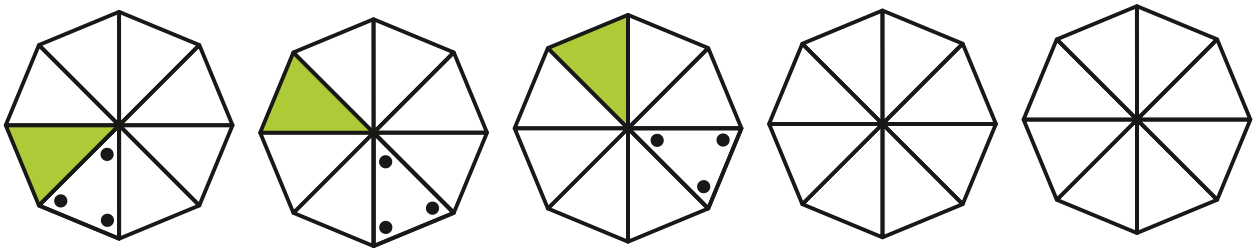
Is it possible that someday Barney will have fish in just one color ?
Explain your answer !

6 Figurative Patterns

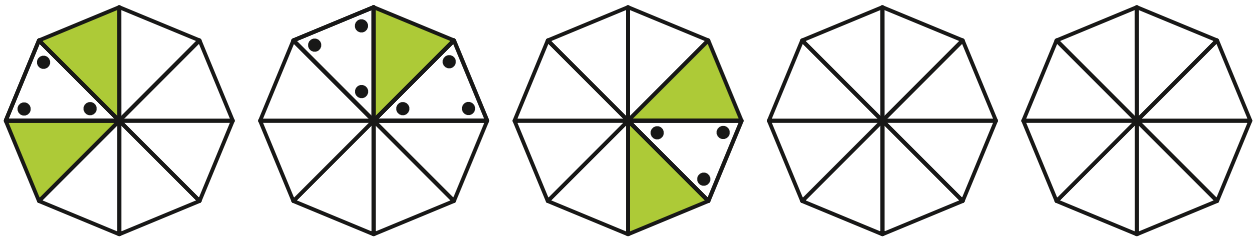
Logical Deductive Reasoning on the Basis of Figurative Patterns

Name: _____

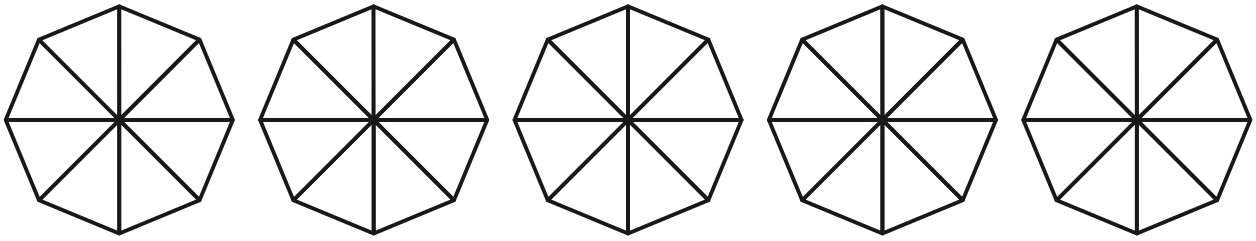
1 **Martin likes to think up patterns.** Continue his pattern !



Also continue Martin's next pattern !



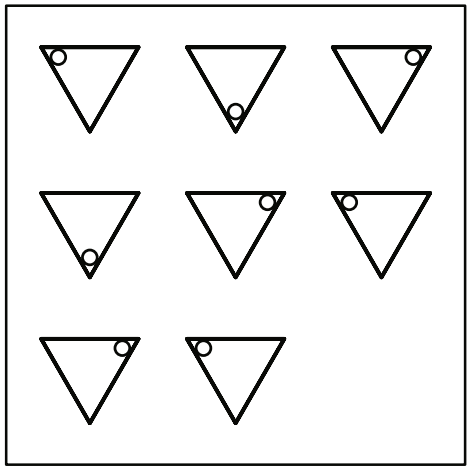
Think up a difficult pattern for yourself !



Why is your pattern difficult ?

Name: _____

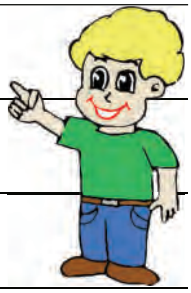
❷ **Till started a special drawing.**



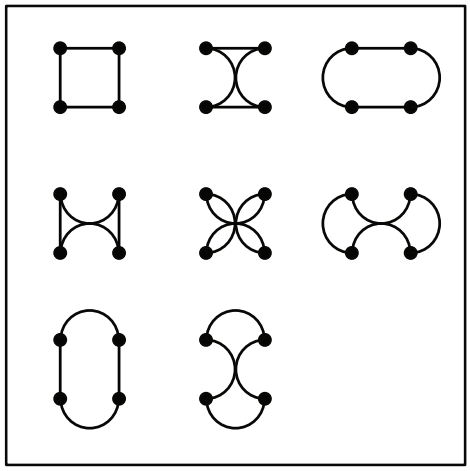
Unfortunately, he was disturbed and did not finish it.
He planned to draw something at the bottom right.

Please finish his drawing.

Explain your solution !



Anna also started a special drawing.



Unfortunately, she was disturbed as well and could not finish it.
She planned to draw something at the bottom right.

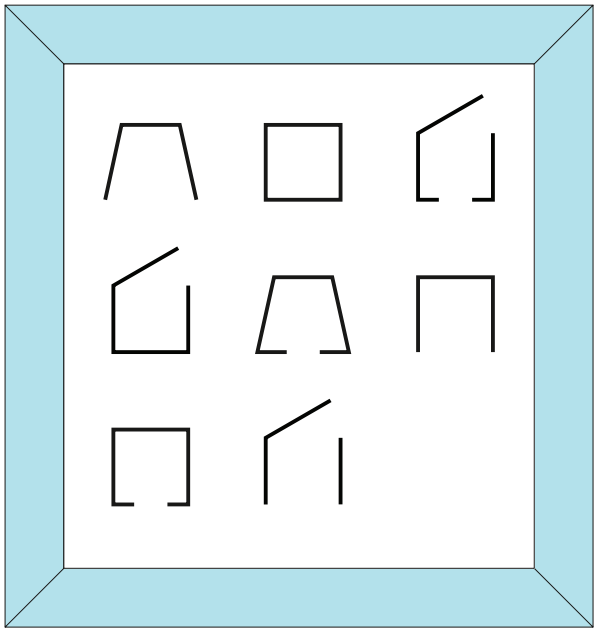
Please finish her drawing.

Explain your solution !

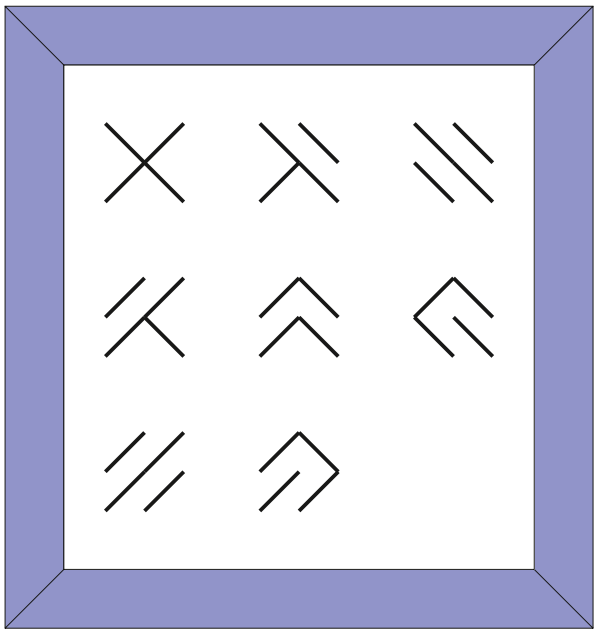
Name: _____

❸ **Elves don't need keys for their door locks.**
The doors open when a riddle is solved.

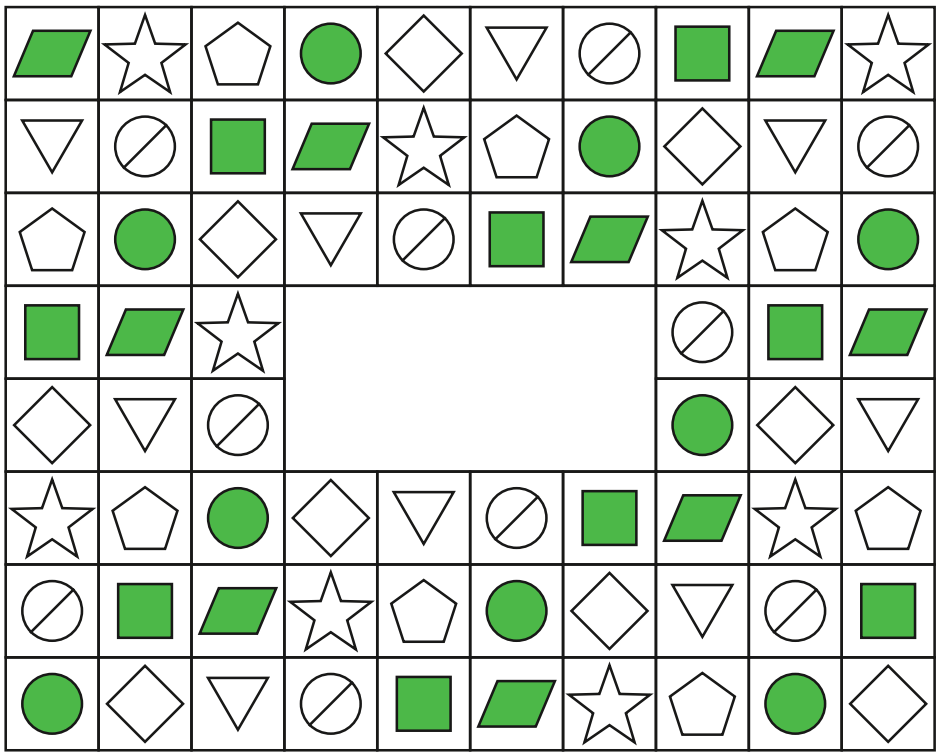
Try to open this door !
Tip: There is something missing in the bottom right.
Draw in what you think is missing !
Explain why the door should now open !



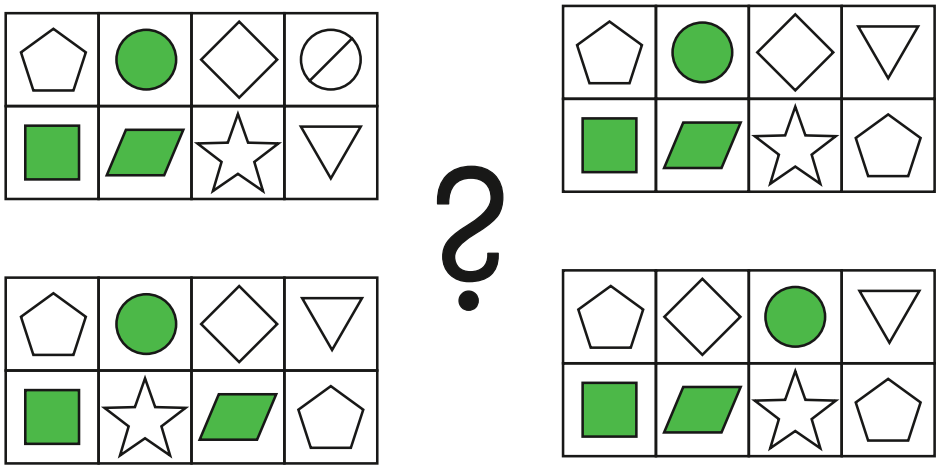
Try to open this door !
Tip: There is something missing in the bottom right.
Draw in what you think is missing !
Explain why the door should now open !



4 **Look closely: there is a gap in the middle.**

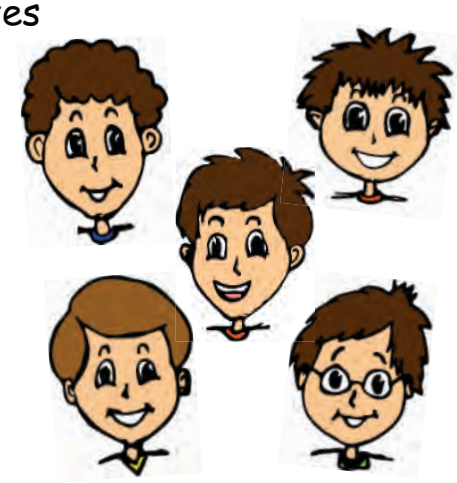


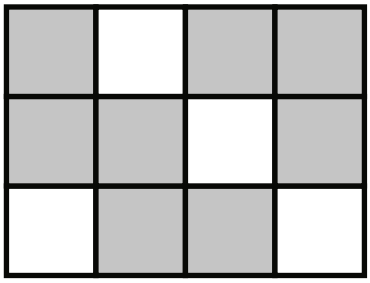
Circle the rectangle that in your opinion fits best into the middle.

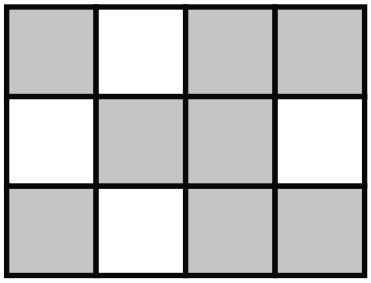


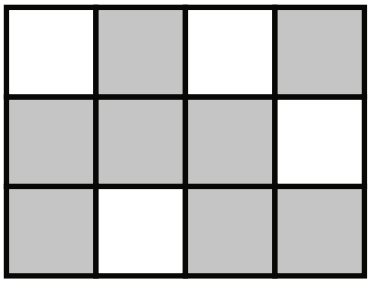
Why does the rectangle you chose fit in well ?

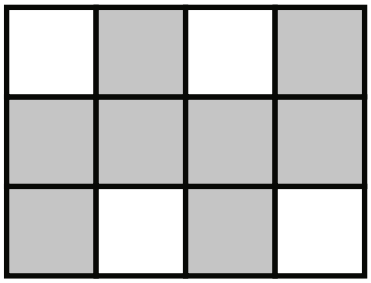
❺ **5 brothers are playing with small plates.** The plates are white on the one side and grey on the other. They lay out 5 patterns.

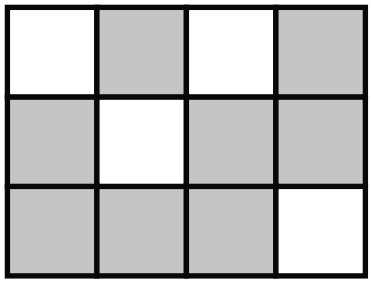


1) 

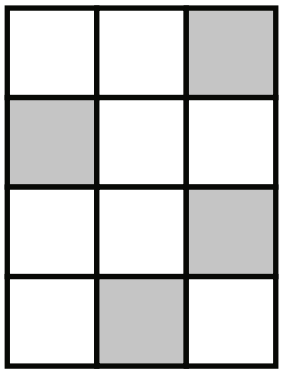
2) 

3) 

4) 

5) 

Which of their patterns fits best to this picture ? _____



Explain your answer.

_____

_____

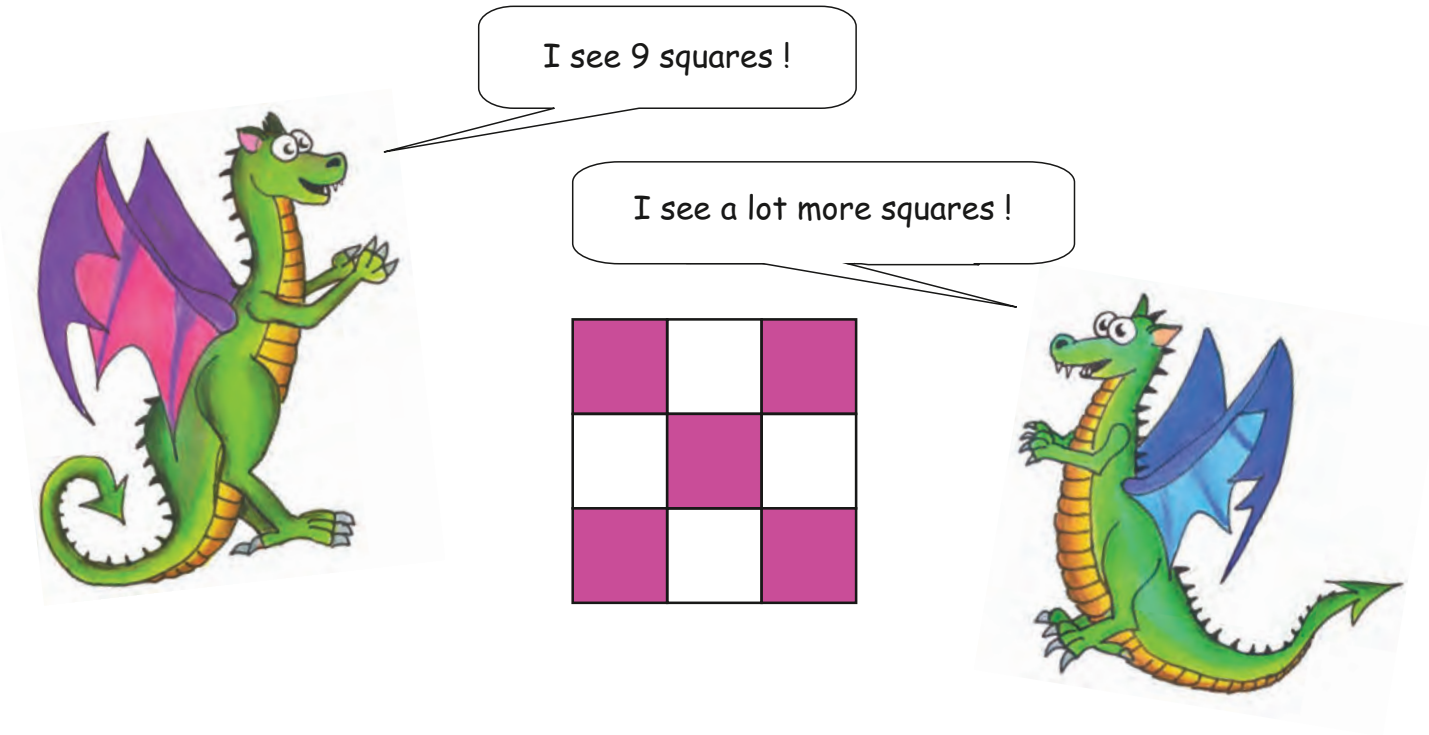
_____

_____

_____

Name: _____

1



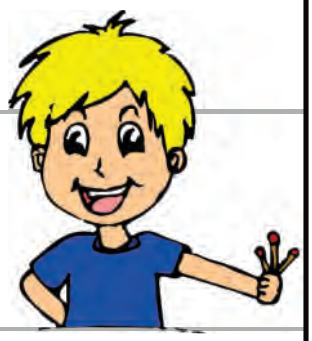
How many squares of any size do you see ?

Answer: _____

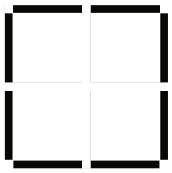
Explain ! Write or draw !

Name: _____

❷ **Tim has some matches.** He wants to use them to lay the 4 sides of rectangular figures. He starts with 4 matches.
Like this he can lay just 1 such figure.



He takes 8 matches.
Now he can lay 2 different rectangular shapes.



Then he takes 18 matches.
How many rectangular figures can he lay now ? _____ Draw them all.

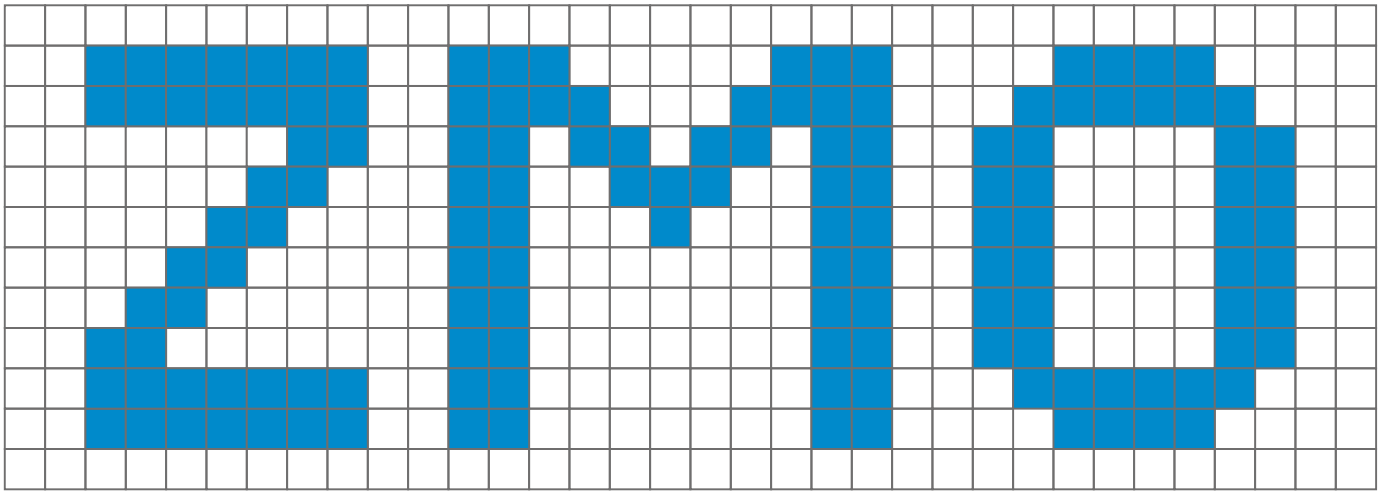
What do you notice ? _____

_____

_____

Name: _____

1 **Tina draws three letters onto squared paper:**



For which letter did Tina have to paint the most squares ? _____

For which letter did Tina have to paint the least squares ? _____

That's how I found my results:

_____

_____

_____

_____

_____

_____

_____

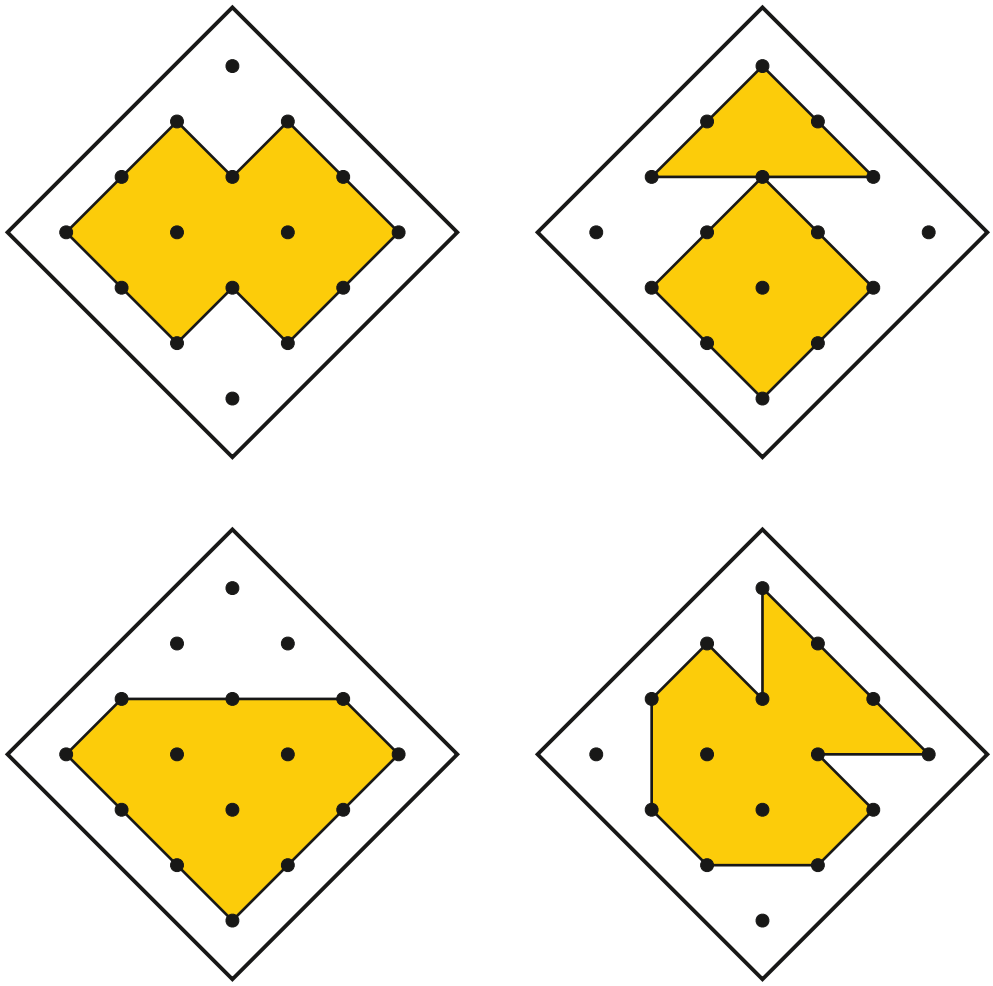
_____

_____

_____

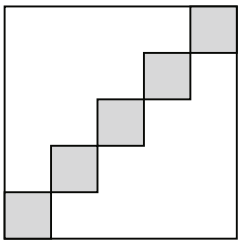
Name: _____

❷ **Examine the areas of the yellow figures !**



What do you notice ?

❸ **Tara has 150 small grey squares.**
She uses them to lay out two areas.
Consider how many squares she needs for each of the areas!
Explain your answers!



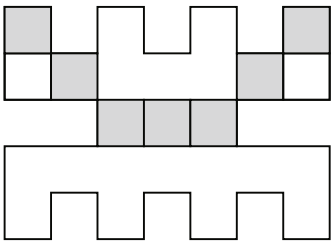
For this, Tara needs _____ small squares.

Explanation: _____

_____

_____

_____



For this, Tara needs _____ small squares.

Explanation: _____

_____

_____

_____

What do you notice ?

_____

_____

_____

_____

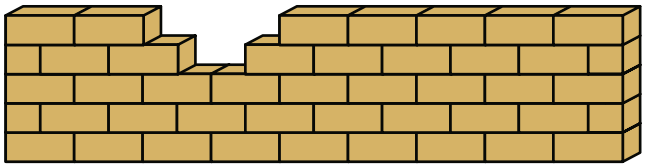
_____

Name: _____

4 **The thievish dwarf.**

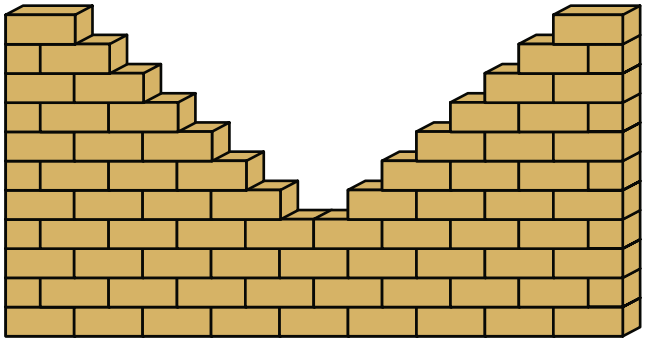
The dwarf stole three bricks from this wall.

But he still needs more bricks.



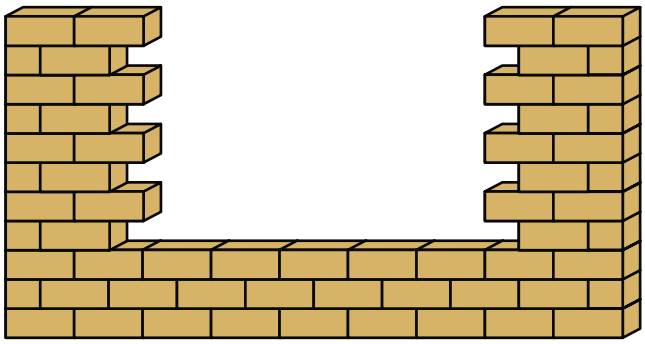
How many bricks did the dwarf steal from this wall ?

Explain your answer !



How many bricks did the dwarf steal from this wall ?

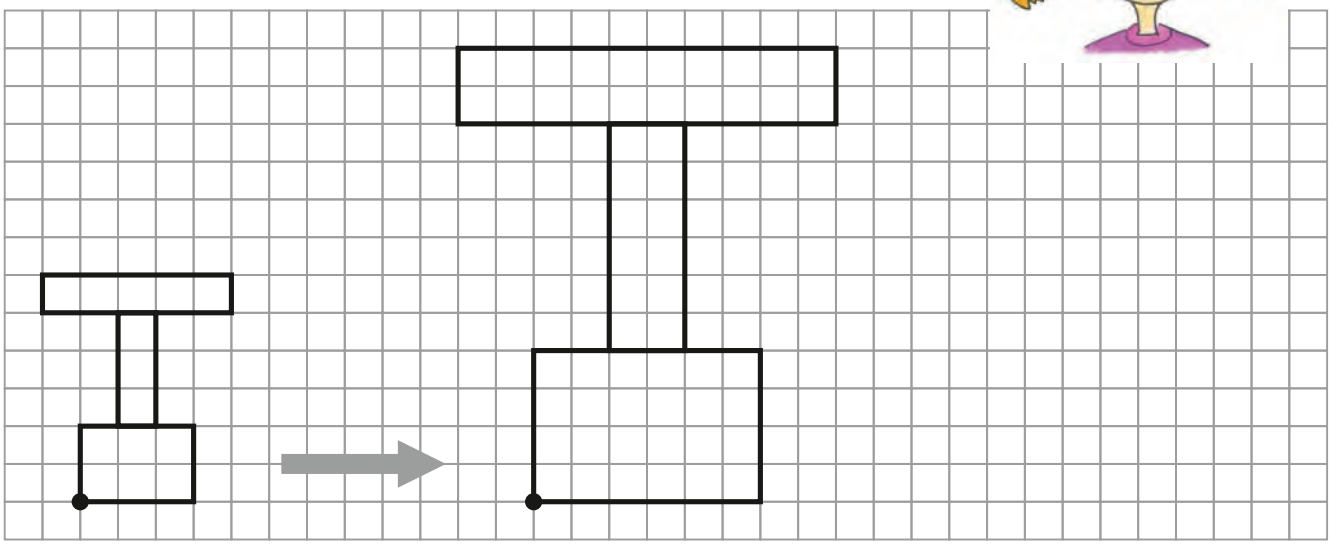
Explain your answer !



Name: _____

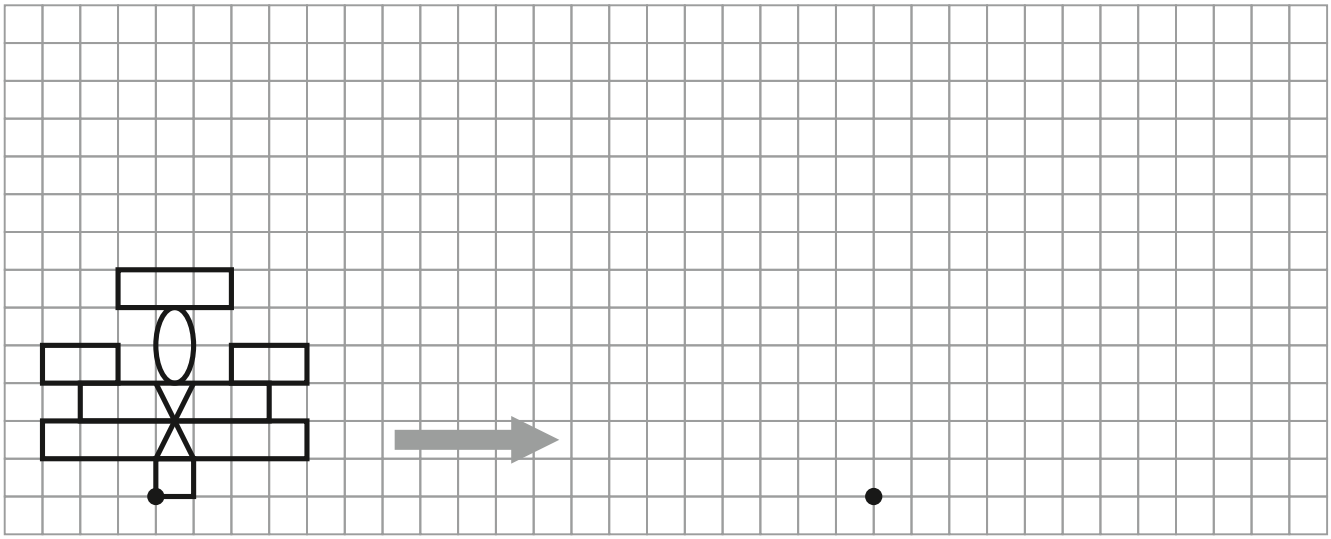
1 **Zena likes to draw.**

Today she practices scaling up.



Zena drew a new figure.

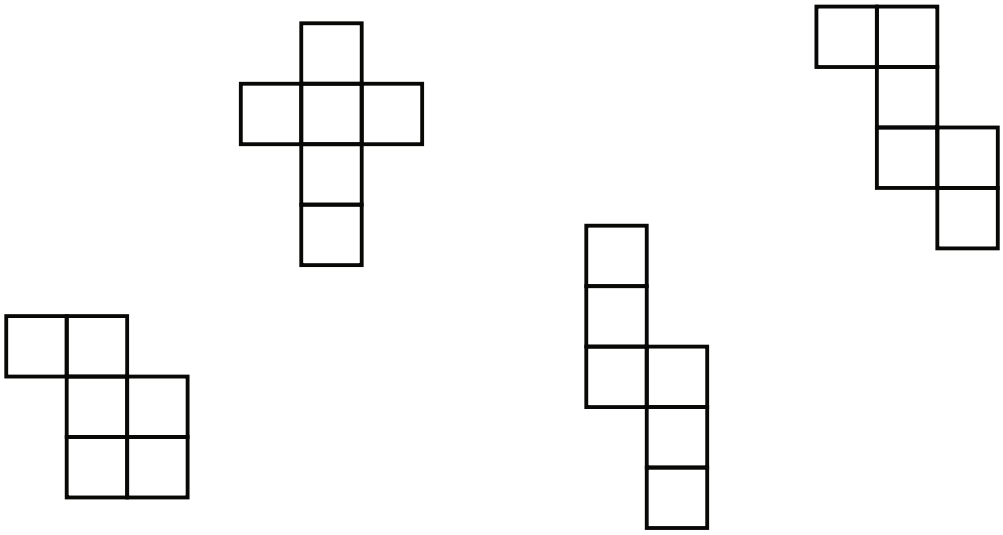
Scale up this figure like Zena did just now !



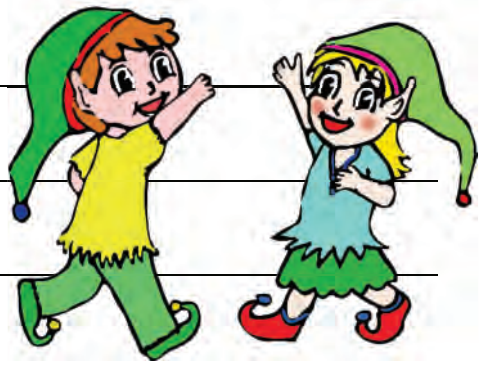
How did you manage the scaling up ?

Name: _____

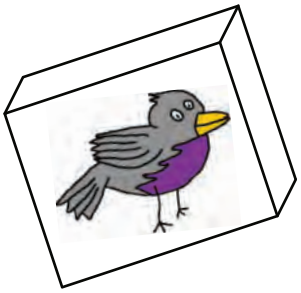
1 **On their party, the elves want to play with dice.** They have templates that they can use to create dice. Look closely ! Which of their templates are well suited to create dice ?



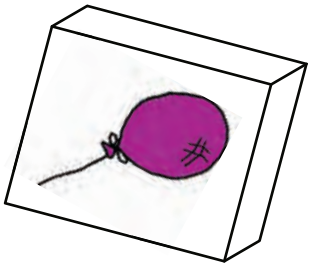
Explain !



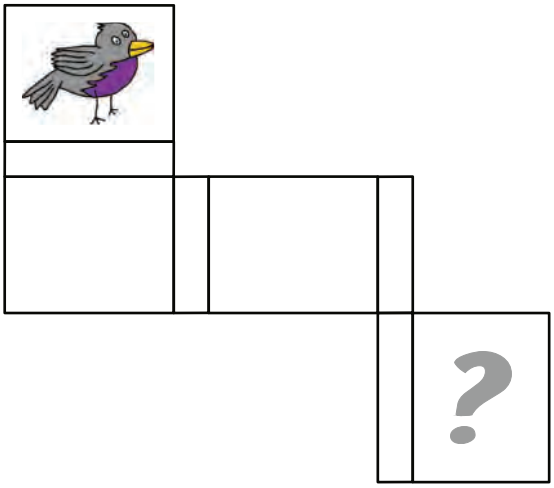
❷



Carlos plays with a box.
On the frontside is a picture of a bird.
On the backside is a picture of a balloon. Carlos carefully tips the box over the paths.

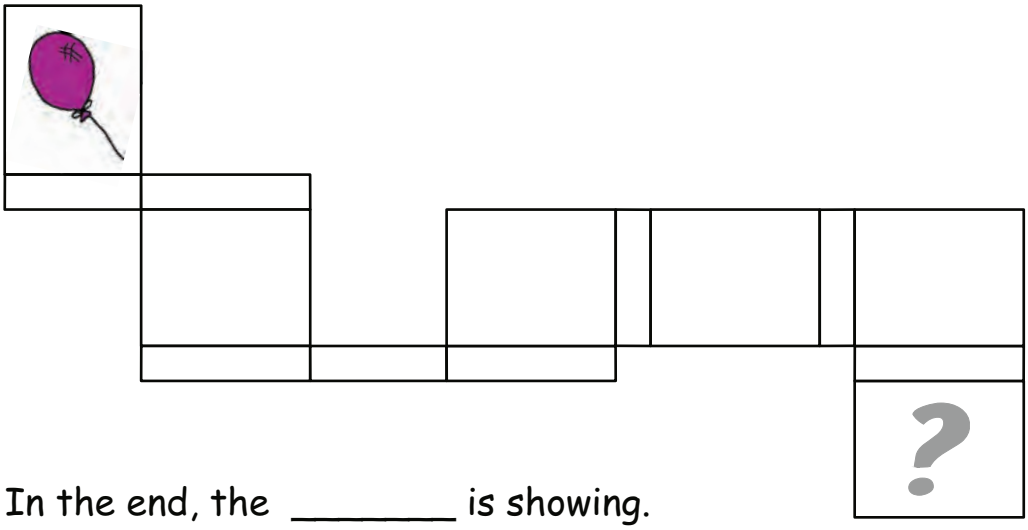


Find out whether the bird or the balloon is showing in the end !



In the end, the _____

is showing.



In the end, the _____ is showing.

What did you notice ?

_____

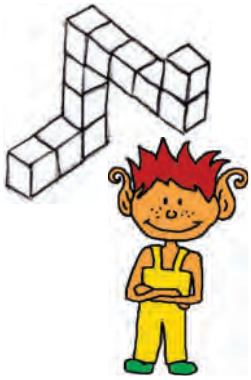
_____

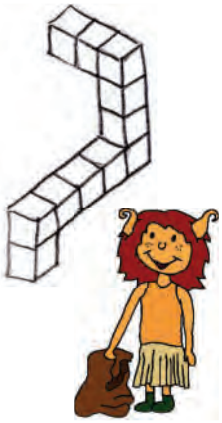
_____

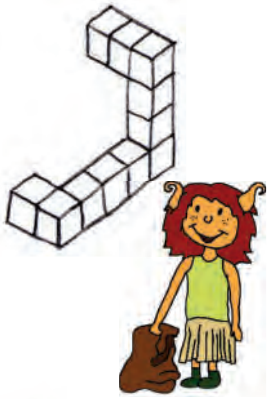
_____

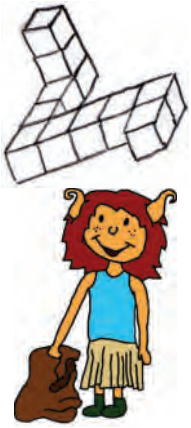
Name: _____

❸ **Fjaro created the same figure as his sister.**
Who is his sister? Draw a line between the two of them !









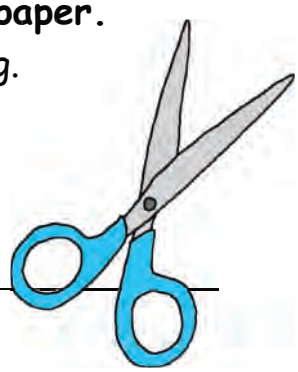
Explain your decision !

Name: _____

❶ **Take a folded piece of paper. Now cut out a figure in such a way that you see a circle when unfolding the paper.**
Think carefully. You may not cut again after unfolding.

Give a hint ! What is important while cutting in order to get a circle as a result ?

❷ **Take a folded piece of paper. Now cut out a figure in such a way that you see a five-pointed star when unfolding the paper.**
Think carefully. You may not change the form after unfolding.



Explain what you considered in order to actually create a five-pointed star.

Name: _____

4 **Carpenter Will created a large wooden die.**

He paints it from all sides with blue color.

How many blue sides does the die have ?

_____

_____

One week later he once again occupies himself with his blue wooden die. He saws it up and receives 27 small wooden dice that are all of the same size.

How did he manage that ?

Will examines his small wooden dice.

How many of them are blue from all sides ? _____

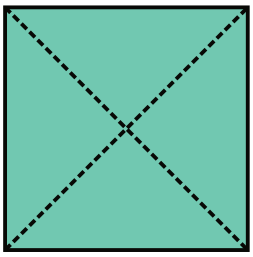
How many of them are blue on only one side ? _____

How many of them are blue on several sides ? _____

Explain your answers ! Calculate, draw or write something down.

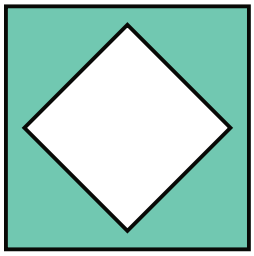
Name: _____

❸



Take a paper and cut it to a square.
Afterwards, fold this square twice along the fold lines you see to the left so that you get a triangle.

Now cut in a way that you receive this after unfolding:



Think carefully ! Don't waste paper.

What is important while cutting ?

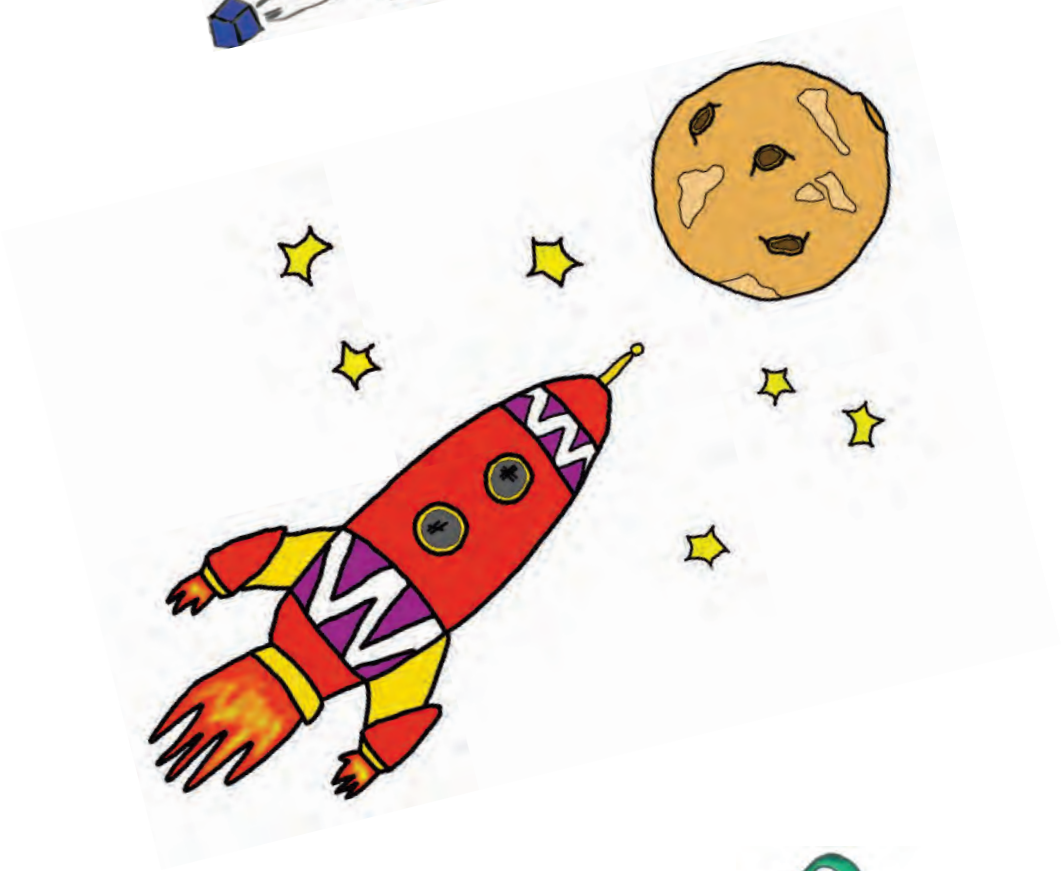
Which tips do you have for other children ?

Name:

Even more space for your thoughts !

7 All's well that ends well

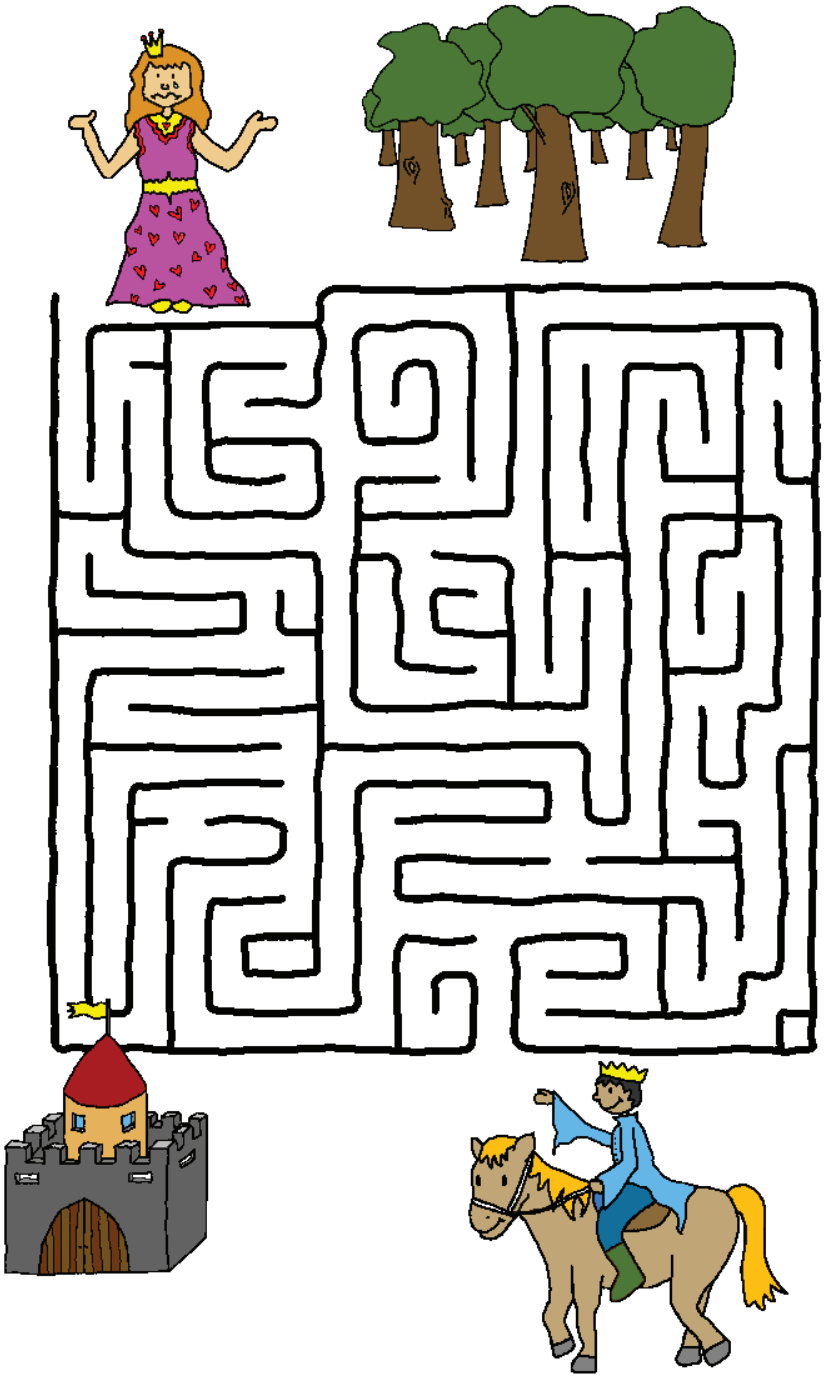
Mazes and more



Name: _____

1 **Help the prince to reach the princess !**

Draw in a convenient path through the maze.



Why is your path convenient for the prince and the princess ?

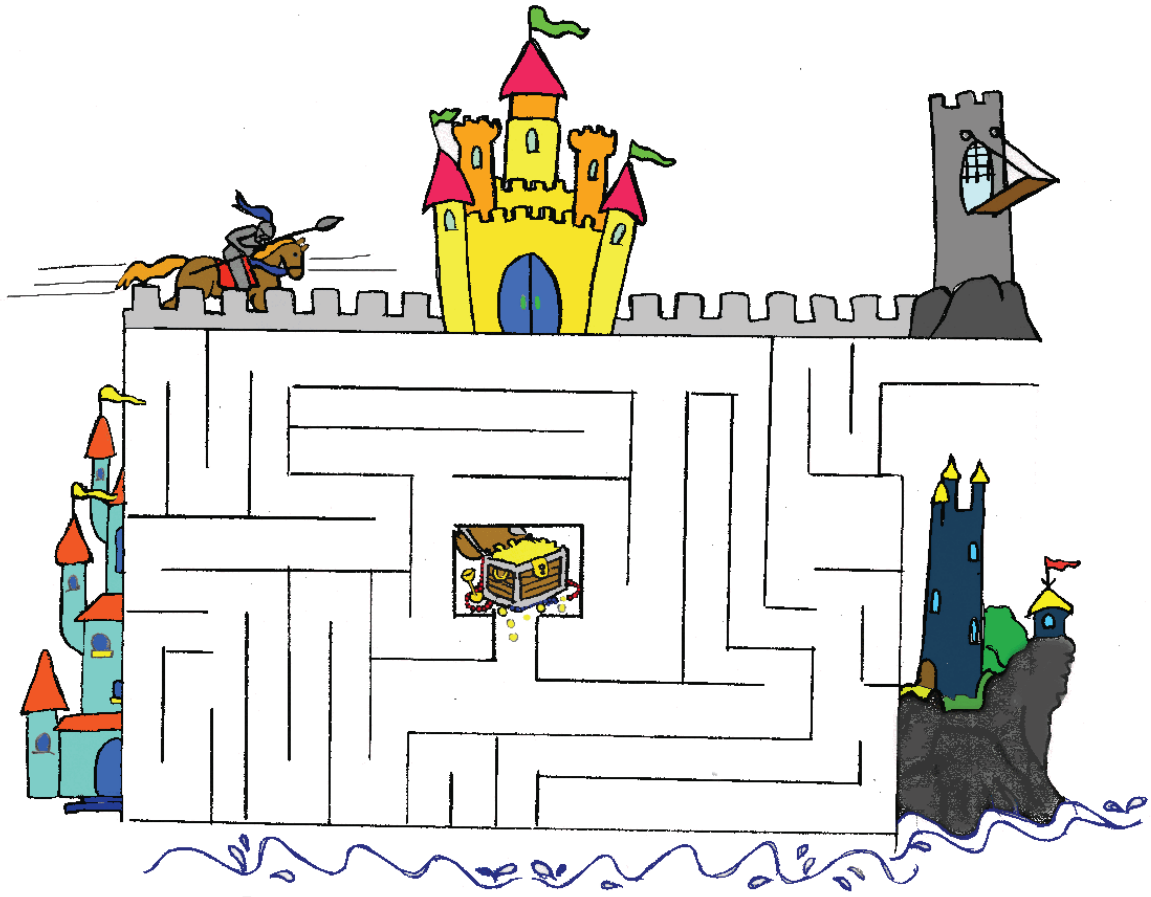
_____

_____

_____

Name: _____

❷ **Find the way to the treasure !**



How can one design the maze easier or more complicated ?

Name: _____

❸ **Think carefully which very hard calculation tasks you can tackle and crack.** Write them down and solve them!

Why is your task a difficult one ?

_____

_____

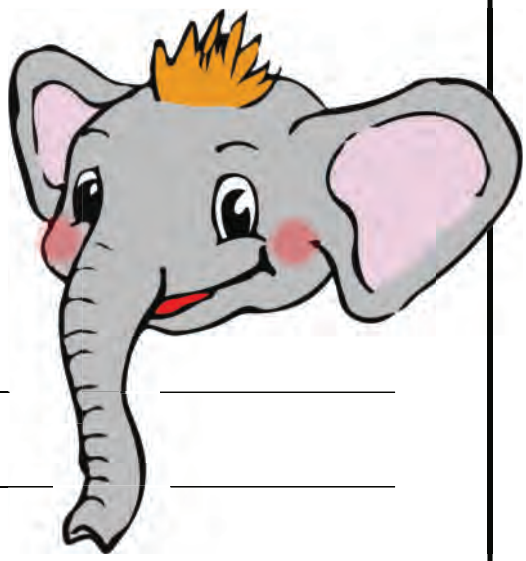
_____

_____

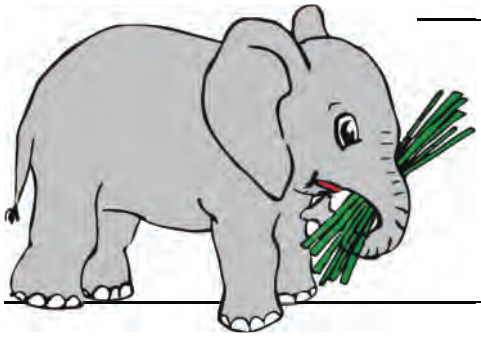
_____

Name: _____

4 **Continue — as long as you like !**



837, 854, 840, 857, _____



Name:

Even more space for your thoughts !

C. Future Perspectives

Math is not a spectator sport.
It's not a body of knowledge, it's not symbols on a page.
It's something you play with, something you do.

~ Keith Devlin ~

In the analysis of the approaches and solutions provided by the best performing children of our sample it becomes apparent again and again that a key element for mathematical understanding is logical-deductive reasoning achieved by thinking in networks and examining mathematical relationships.

Besides our engagement concerning mathematically gifted children, we also set great store by our long-term research about the development of mathematical thinking including the examination of basic principles of logical thinking. In this context, we adapted one of Spearman's task formats (Spearman, 1904; see also Raven, 1965) and specified it in terms of two different types of inferential logical reasoning. The fundamental idea behind this is that depending on the basic thinking procedures a person uses, the orientation in the world, types of sources for getting insight and the personal construction of a world view are not the same. Let us examine one of our matrices (Figure D1). In the bottom right corner there is a figure missing. Which one might fit in? Why?

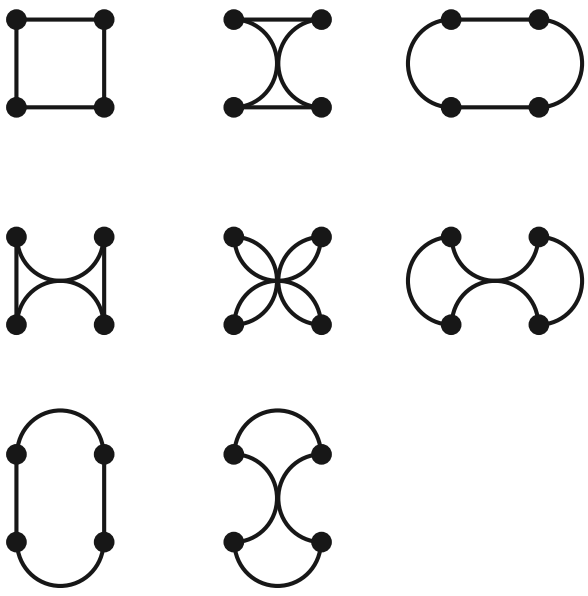


Figure D1: Task example from the Qualitative Diagnostic Instrument for Predicative-logical vs Functional-logical Thinking (QuaDiPF, (Schwank, 1999/2000)

Cognitive procedures allow establishing relations between the figures: specific figure components come to the fore while other components remain apparitional. The mechanism determining which components are focused on and which others remain excluded is the basis of the two thinking types we defined:

Predicative-logical thinking: A type of inferential logical thinking that establishes relations by focusing on similar or identical components of objects. Concerning the given task, in the first row the upper and lower lines 'belong together', but the side lines do not fit the pattern (Figure D2). The method of focusing on what belongs together also works in the remaining rows. This categorization generates order and structure. Continuing this cognitive appraisal results in a figure with lines from the bottom row (upper and lower) and the right-hand column (sides) respectively.

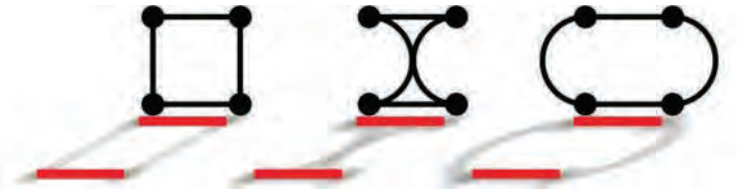


Figure D2: Mental construction with Predicative-logical focus
Upper and lower lines that are noted to be identical.

Functional-logical thinking: A type of inferential logical thinking that establishes relations by thinking in action sequences and achievable effects by means of what differences in objects can be converted into one another (Figure D3). Concerning the given task, in the first row the side lines can be pulled inward and subsequently outward. This mentally creates a dynamic view of alterable figures. Continuing this cognitive appraisal results in a figure with the upper, lower, and side lines pulled outward. Actor-bound reasoning is required in order to produce result figures (Figure D4).

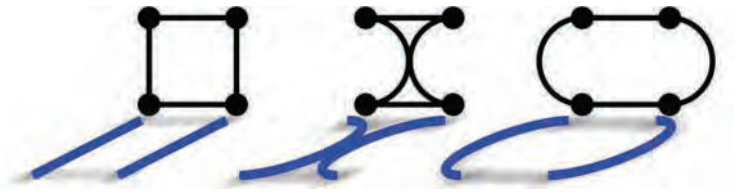


Figure D3: Mental construction with Functional-logical focus.
Lateral lines that are first pulled inwards, then outwards.

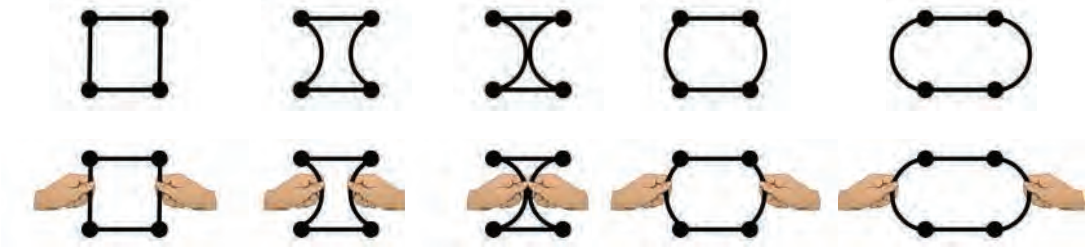


Figure D4: By means of action-oriented reasoning given target figures can be created.
In a figurative sense, hands on is essential for initiating and controlling the process.
Just passively observing is not sufficient.

In line with the results from other psychological studies, our analyses revealed that boys tend to be stronger in functional-logical thinking than girls (cf. Schwank, 2001, 2003). Concerning early mathematics education which is strongly coined by arithmetic content, this difference has to be taken into account. Calculating means handling numbers. Numbers that are changed while calculating, in different manners depending on the specific calculation method used. The objective is to reach results. An understanding of the numbers themselves as emerging from calculation processes facilitates this.

The art in teaching an understanding of numbers therefore lies in providing children with adequate arithmetic experiences that demonstrate the action-based connections between numbers. This would require math lessons to put a greater emphasis on the experiencing of and reflecting on functional connections. For this purpose, actions guided by playful rules are indispensable. Didactical material needs to be freed from out of the shadow of its existence as a mere supportive auxiliary and receive attention and significance: As playworlds allowing the flourishing of mathematical thinking by practically demonstrating mathematical problems and their possible solution approaches. Fixation on the written language of mathematics bears the risk of a dangerous simplification of thinking processes merely consisting of a static object view based on memorized knowledge instead of truly grasping and applying the functions of numbers and arithmetic operations in terms of a process view (also see Hefendehl-Hebeker, 2001; Schwank & Nowinska, 2008).

For several years, our Meeting Point has made strong efforts to develop mathematical playworlds allowing the implementation of exactly this type of math lesson so urgently needed. Up to now we created the ENSO (event-related number space orientation) playworld for exploring the number range from zero to nine, the Spiral Stairs of Calculation for accessing the number space from 0 to 19, the Stellanian Accounting System for reaching an understanding of the functioning of the place-value system and the Number Skyscraper for facilitating a multiplicative understanding (e.g. lcm, gcd, square numbers, binominal formulas), thereby building a strong action-oriented foundation for early mathematics education generating a process-based mathematical understanding. From preschool onwards, the Dynamic Labyrinths offer a playful introduction into a basic concept formation necessary for a mathematical understanding of automatization and programming (for further information see http://www.ikm.uni-osnabrueck.de/aktivitaeten/dl/dynamic_labyrinths/dynamic-labyrinths.html).

All the material mentioned above has been successfully put into practical use in countless lessons with children ranging from kindergarten to secondary school age as well as trainings for educational professionals. The unanimous approval from both adults and children give rise to the hope that in the near future, these educational methods will lead to a profound, process-oriented mathematical understanding for many more children resulting in enjoying the active and competent exploration of the mathematical world.

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Schwank, I. & Nowinska, E. (2008). Die Denkform des Formeldenkens. [The Thinking Form of Formula Thinking.] In B. Barzel, T. Berlin & A. Fischer (Hg.):Algebraisches Denken. Festschrift für Lisa Hefendehl-Hebeker. [Algebraic Thinking. Commemorative Publication for Lisa Hefendehl-Hebeker.] 111-122. Hildesheim: Franzbecker.

D: On a final note

Certificates (Copy Templates)

Plaom-Certificate

Bronze-Certificate

Silver-Certificate

Gold-Certificate

Certificate for teachers

ZMO-Team-Members
Without their support the ZMO could not have been performed for a period of 13 years.

Statements of the ZMO-children concerning participating at the ZMO-BrainTrain-Round

Big yearly final ZMO-celebration at the University of Osnabrück

Zingy Mathematical Olympiad

Name

Primary School

Class

Zingy Mathematical Olympiad

BRONZE

Name

Primary School

Class

Zingy Mathematical Olympiad

SILBER

Name

Primary School

Class

Zingy Mathematical Olympiad

GOLD

Name

Primary School

Class

Zingy Mathematical Olympiad

At the primary school

_____

the class / advanced eduaction group

_____

has been intensively and successfully guided by

_____

The ZMO-Team-Members

Continuous scientific direction: Prof. Dr. Inge Schwank

Thanks to the great cmmitment of many, it was possible to conduct the ZMO for 13 years. Some of them are true ZMO-fans and participated in many rounds.

Most members were students majoring in elementary school education with the subject Mathematics, some other were University employees or firmly convinced volunteers.

All of them deserve our many thanks!

Zingy Mathematical Olympiad

for 3rd graders in the town and district of Osnabrück

XXIX

Anke Aring

Alexander Auch

Kerstin Bartke

Anke Becker

Verena Beckmann

Carsten Beernink

Chantal Bennek

Kathrin Blocksdorf

Katja Boeck

Miriam Bollmer

Imke Bolz

Sina Böttger

Jana Bröcker

Bianka Bruchwald

Lisa Brückel

Frauke Bruns

Prof. Dr. Elmar Cohors-Fresenborg

Anna Deppen

Carina Deters

Julia Detert

Johannes Dieker

Nicole Dijks

Susanne Dreier

Verena Dübbert

Annika Düvel

Christine Eckseler

Sabrina Frieling

Wiebke Fritz

Kathrin Fühner

Marion Gawlik

Christopher Gerke

Regina Gerlach

Katharina Gleis

Eva Maria Gretzmann

Corinna Hänisch

Dorit Heckeroth

Franziska Heckeroth

Dana Heinze

Vanessa Hermes

Marianne Herzberg

Wibke Hille

Manuela Hilmes

Burgis Hoffmann - zu Höne

Pia Hörstermann

Martina Hülsmeier

Thea Israel

Solveig Jensen

Sabine Jones

Emilia Jüngling

Angie Kalverkamp

Marei Kaminski

Katharina Karrasch

Jörg Erik Kinner

Dennis Klaffei

Wiebke Klaue

Anja Knochenwefel

Anne Köhler

Judith Koonen

Natascha Korte

Kathrin Kraicziczek

Björn Kremp

Margit Krützhake

Jenny Kursawe

Thomas Kybart

Maria Lager

Judith Lagies

Eva Lasar

Nils Linnemann

Johanna Lohmann

Nicole Lüdiger

Gaby Lüken

Monika Lütke Dreimann

Tomke Lüttel

Julia Lux

Sabrina Macke

Corinna Maier

Fabienne Martini

Annalena Masur

Ramona Vanessa Meier

Bastian Mertens

Tanja Meyer

Bin Müller

Nina Müller

Edyta Nowinska

Katrin Pech

Marlen Petermann

Stefanie Petersen

Lutz Picht

Stefanie Plagemann

Gabriele Plietz

Melanie Ploppa

Inka-Maria Pohl

Maren Pötter

Torsten Pretschner

Frank Pundsack

Franziska Quade

Bianca Raddatz

Philipp Rahe

Karen Räsch

Ina Ricker

Cornelia Riepe

Jörg Ritterbusch

Tina Rohde

Florian Röhrs

Anuschka Ruge

Irina Ruks

Petra Sandeck

Diana Schall

Christina Schaper

Diane Schemme

Moana Schilberg

Thomas Schinkowski

Annedore Schmidt

Johanna Schmidt

Kathrin Schnalle

Stefan Schneider

Mareike Schnoor

Britta Schomakers

Janina Schröder

Elisabeth Schwank

Julia Sibbing

Ina Spreen

Anna-Luise Strauch

Kristin Straukamp

Katharina Tarp

Anne Theisling

Irene Thiemann

Imke Tjaden

Nadine Träger

Sophia Varnholt

Anna-Bianca Viebrock

Julia Vischer

Alla Vlasenko

Dirk Vorhold

Carena Wellmeyer

Anja Wemhoff

Corinna Wendeln

Sonja Weßel

Christine Wiechers

Maren Wies

Julia Winckler

Josephine Windisch

Kirsten Winkel

Inga Winkelmann

Christina Woitschek

Wera Wortelen

Philipp Zumdohme

Janina Zwirner

Statements of the ZMO-children

We want to participate in the ZMO-BrainTrain-Round because...

wir herausgefordert werden möchten.
we want to be challenged.

weil's Spaß macht!
Because it's fun!

ich Mathe sehr sehr gerne mag und ich werde alles geben. Ich war ganz aufgeregt als ich ausgewählt wurde. Irgendwie war mir das sehr wichtig. Das Gefühl kann ich nicht beschreiben. Ich bin sehr glücklich.
I like math very very mich and will do my very best. I was totally excited when I was picked. Somehow this was important to me. I can't describe that feeling. I am very happy.

ich zeigen möchte was ich alles kann.
I want to present my abilities.

wir schon so viel in Mathe gelernt haben und wir dann bestimmt gute Chancen haben zu gewinnen.
we have learned so much in math, therefore our chances of winning are certainly good.

Weil wir Mathe gut finden. Weil wir gerne etwas neues lernen wollen.
Because we like math.
Because we want to learn something new.

Damit wir unseren Kopf mal richtig einschalten können.
So that we can properly switch on our heads.

wir für die ganze Klasse mitmachen

we participate for the whole class

Weil ich gerne Mathearbeiten mag. Ich möchte gerne testen wie gut ich wirklich bin.

Because I like math tests. I want to test how good I really am.

wir Schwierige Aufgaben mögen, gerne knobeln und Spaß an Mathe haben.

we like difficult tasks, puzzling and having fun with math

es etwas besonderes ist.

it is something extraordinary. it is very nice that we can do this.

es ist sehr schön das wir es dürfen.

Weil ich Mathe liebe. Und wenn man dann auch noch Preise gewinnen kann ist das Super.

Because I love math. And that it is also possible to win prizes is great.

... wir gerne unsere Klasse gut vertreten möchten.

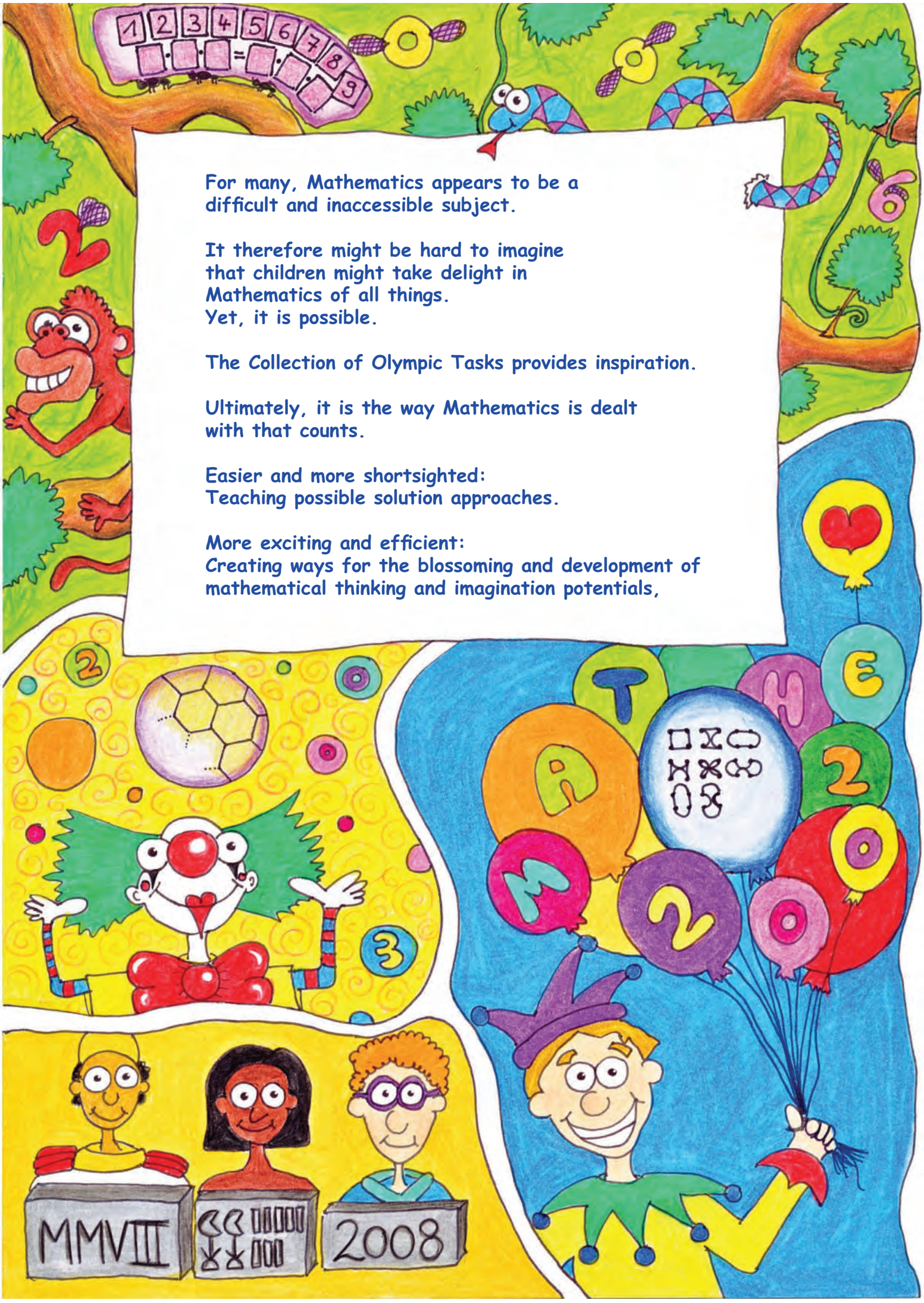
... es sicherlich viel Spaß machen wird, mit anderen Kindern zu rechnen.

... es ein spannendes Erlebnis sein wird.

...we want to represent our class well.
...it will surely be great fun to calculate together with other kids.
...it will be an exciting event.

wir Mathe mögen und in der Hirnsportrunde kann man mal so richtig den Kopf anwerfen.

we like math and in the BrainTrain-Round it is possible to properly switch on one's head.



For many, Mathematics appears to be a difficult and inaccessible subject.

It therefore might be hard to imagine that children might take delight in Mathematics of all things.
Yet, it is possible.

The Collection of Olympic Tasks provides inspiration.

Ultimately, it is the way Mathematics is dealt with that counts.

Easier and more shortsighted:
Teaching possible solution approaches.

More exciting and efficient:
Creating ways for the blossoming and development of mathematical thinking and imagination potentials,